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Process-Oriented Routines of Students in Heterogeneous Field Dependent-Independent Groups: A Commognitive Perspective on Solving Derivative Tasks

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Abstract: Students are more likely to obtain correct solutions in solving derivative problems. Even though students can complete it correctly, they may not necessarily be able to explain the solution well. Cognition and communication by the students will greatly affect the subsequent learning process. The aim of this study is to describe students' commognition of routine aspects in understanding derivative tasks for heterogeneous groups of cognitive styles-field dependent and independent. This qualitative study involved six third-semester mathematics education students in the city of Palu, Indonesia. We divided the subjects into two groups with field-independent (FI) and field-dependent (FD) cognitive styles. The first group consisted of two FI students and one FD student, and the second group consisted of two FD students and one FI student. Moreover, the subjects also have relatively the same mathematical ability and feminine gender. Data was collected through task-based observations, focused group discussions, and interviews. We conducted data analysis in 3 stages, namely data condensation, data display, and conclusion drawing-verification. The results showed that the subjects were more likely to use routine ritual discourse, namely flexibility on the exemplifying category, by whom the routine is performed on classifying and summarizing categories, applicability on inferring category, and closing conditional on explaining category. The result of ritual routine is a process-oriented routine through individualizing. This result implies that solving the questions is not only oriented towards the correct answers or only being able to answer, but also students need to explain it well.

Keywords: *Cognitive style, commognition, derivative, heterogenous groups, routines.*

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Introduction

Learning is always seen as a process of change. In the 20th century, there were disagreements about what changes when learning takes place. Student behavior will change by learning according to behaviorists. Simultaneously, the cognitive view conceptualizes learning as a process of acquiring knowledge and changing mental structures. Furthermore, in the theoretical framework commognition learning mathematics is defined as changes in students' discourse that can be seen when they communicate (Tabach & Nachlieli, 2016).

Piaget (1952) states that the direct interaction of individuals and the world will result in intellectual growth. This opinion is explicitly opposed by Vygotsky, who states that whatever a person is learning, whether she uses the term of knowledge, concept, or higher mental function, will refer to a culture that is produced and modified by collective human efforts. Furthermore, Sfard (2008) stated that learning is legitimate peripheral participation, with its basic principle is a patterned collective form. Actions and reactions repeatedly occur during the communication activities. Every repetition occurs only in certain parts that can be observed, namely a pattern. One example of a pattern is the initiation – response–evaluation (IRE) which is a characteristic feature of teaching and learning interactions. According to Duval (2006) and Sfard (2008), mathematics activities rely heavily on the use of different semiotic systems such as mathematical words, algebraic symbols, graphs, and pictures.

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Thinking (cognition) is an inherent individual activity that develops from a patterned collective activity. The collective activity that turns into thinking through a process of individualization is called interpersonal communication. According to Sfard (2008), human thinking can be considered as a personal form of communication activity, namely communication with oneself. Self-communication does not always mean being able to be heard or seen, and it does not have to be in words. Furthermore, Sfard states that thinking (cognition) is a dialogical sense when we provide information, ask questions, arguing, we wait for information and responses from others. As a result, thinking and communicating cannot be separated.

Sfard (2008) combines the terms of cognition and communication into a new adjective, as commognition. The cognitive paradigm explicitly describes the activities of mathematics, thinking, and learning. Commognition is rooted in collective activities that follow certain rules, which refer to the objects and media used. According to Wittgenstein (1953) rules look like a game. A game plays with a variety of tools and must comply with applicable rules.

Sfard (2008) defined learning as a commognitive activity that includes reasoning, abstracting, objectifying, subjectifying and consciousness. The reasoning is the systematic derivation of utterances from other utterances. Abstracting is creating and communicating objects that are intangible or concrete. Objectifying is substituting a noun for a process that does not involve a human actor. Subjectifying focuses on the human being who performs the activity. Consciousness involves thinking about thinking – the ability to act either as an actor or as a judge.

Mathematical discourse is the main object of commognitive research, and which is why the commognitive distinguishes from other studies. As a form of communication activity, learning is understood as a phenomenon that is inherently collective or social and not individual. Discourses are a type of communication different from commognition that unites several individuals and excludes some other individuals. Mathematical discourse includes a unique way of saying and doing, which can be identified with the help of four characteristics of discourse: keywords, visual mediator, endorsed narratives, and routines (Tabach & Nachlieli, 2016).

Keywords are a very important issue that is called "a word meaning." Visual mediators are how discourse participants' instruments to identify the object of conversation and coordinate their communication. A unique visual mediator, for example, is a number, an algebraic symbol, and a graph. Narrative is any text, spoken or written, framed as a description of an object or a relationship between an object or activity with or by an object. Moreover, narrative abides on support or rejection, which is labeled as true or false. Endorsed narratives are known as a mathematical theory which includes discursive concepts, namely definitions, proofs, and theorems. Routines are a feature of discourse due to repetitive patterns. The regular use of keywords and visual mediators in the narrative is referred to as routine (Robert & Roux, 2018; Sfard, 2008). Sfard divides routine into three types, explorations, rituals and deeds. Furthermore, Lavie et al. (2019) divide routines into two, namely, practical and discursive.

Many researches have been conducted on commognition theory in derivative discourse (Lefrida et al., 2020; Nardi et al., 2014; Ng, 2016, 2018; Park, 2013). Moreover, research in the discourse of functions was conducted by (Nachlieli & Tabach, 2012; Tabach & Nachlieli, 2016). Furthermore, Bergqvist (2007) and Tallman et al. (2016) have focused on giving assignments that paid attention to students' cognitive level. Sfard (2020) stated that each mathematical discourse, when taught in a different institutional or cultural environment, could lead to a different learning process.

One of the main goals of learning mathematics is that students will have the right understanding of mathematical knowledge. Hiebert and Carpenter (1996) stated that an internal network can be defined as a mathematical idea, fact, or procedure. In particular, if a network of representations which involves the mental representation of students can be satisfied, then it can be said that students have understood mathematics. An idea or procedure in mathematics that can be understood by students as a person's mental part. This is part of a network that is interconnected with internal representation. In other words, a student can be mathematically understood when their mental representation connects to their mental picture thought. Furthermore, National Council of Teachers of Mathematics (2000) reveals that the student should carry mathematics learning with understanding. This can be conducted by actively developing new knowledge based on their prior knowledge.

The level of students' understanding of mathematics, especially derivatives, is still not satisfactory based on the experience of the leading researcher as a calculus teacher. Many of the students make mistakes in the use of terms, reading symbols, and these errors are often repeated and similar. In addition, students also have difficulty providing explanations about solving the problems they have been working. Cognition and communication play a role in the learning process because one's thinking can be seen by understanding the discourse. Derivative is a basic material that undergraduate students must study at the Mathematics Education Study Program. This material is provided in the first semester which is included in the Differential Calculus course. The primary textbook used in teaching the derivative is the 9th Edition of Calculus written by Varberg et al. (2010), translated into Bahasa Indonesia. Moreover, this course also uses the Calculus Handout written by the teaching team.

To understand mathematics is to construct a network of mental representations of mathematical concepts. Students can be said to conceptually understand when there is some connectivity between the new knowledge gained and their prior knowledge. The understanding category in the cognitive process is divided into seven subcategories (Anderson &

Krathwohl, 2001), namely interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. The description of each of these seven cognitive processes was considered in developing the task for understanding derivative. Furthermore, the questions were then given to the subjects to be discussed in the group. Based on the results of the discussion by each subject on the task for understanding the derivative, we can see the students' commognition.

Students have different characteristics in understanding material that called by the cognitive styles. The cognitive style can influence one's behavior and other processing strategies sometimes depend on task demands (Riding et al., 1993). Furthermore, Armstrong et al. (2012) stated that information processing (analyzing, perceiving, organizing) based on brain mechanisms and cognitive structure is individually different which is more preferred in cognitive style. This study uses the Group Embedded Figures Test (GEFT) cognitive style test instrument developed by Witkin et al. (1971). Subjects who have field-independent (FI) and field-dependent (FD) cognitive styles use the specified criteria. The criteria used in the selection of subjects applied the Kepner and Neimark (1984) criteria, that is, subjects who can correctly answer 0-9 items are classified as dependent fields, and 10-18 items are classified as independent.

This study discusses student discourse in understanding derivatives. The discourse itself cannot be separated from one's thinking activity. Therefore, understanding how someone thinks mathematically can be done by understanding the discourse. Thus, the research results outlined in this article will be more readily understood and used in helping someone learn mathematics.

Methodology

Research Goal

This study describes students' commognition in the routine aspect of understanding derivative tasks in heterogeneous groups of field-independent (FI) and field-dependent (FD) cognitive styles. In this study, we revealed students' commognition by first asking the subjects to solve the task of understanding derivatives in a focused group discussion. Based on the data, we explored the student's discourse in completing derivative tasks and then analyzed it in more detail and deeper. Therefore, this research examined how students' commognition in routine aspects in understanding derivative tasks in heterogeneous groups of FI and FD cognitive styles.

Sample and Data Collection

The research subjects were the third-semester mathematics education students in the city of Palu, Indonesia. Moreover, the subjects were divided into two groups, and each group consisted three students. The first group consisted of two FIs and one FD, while the second group was with one FI dan two FDs. Mathematical abilities are relatively similar in each group. The supporting data collection tools are 1) GEFT instrument developed by Witkin et al. (1971) to determine subject categories based on FI and FD cognitive styles, 2) Mathematical Ability Test (TKM) consisting of 10 essay questions to select the subjects of relatively the same mathematics ability, 3) Existing gender questionnaire, and 4) Tasks of Understanding Derivatives (TMT) based on the seven cognitive processes, namely interpreting, exemplifying, classifying, summarizing, inferring, comparing and explaining. The main data collection proceeded as follows: (1) Giving the task of understanding derivatives to subjects of each group, (2) Observing them during the work, and (3) Interviewing each subject based on the tasks.

Data Analysis

The data analyzed in this study were interview transcripts, subjects' works, and field notes. We followed three stages of qualitative data analysis by Miles et al. (2014). The first stage entailed: 1) selecting data that is relevant to the research objectives, (2) focusing on data that is appropriate for commognitive discourse (keywords, visual mediators, endorsed narratives, and routines), (3) simplifying by making data components not many, (4) abstracting data sets that have the exact nature and characteristics in one concept or category so that attributes are obtained and finally can be formulated, (5) transformation is carried out to understand the data meaning if it is associated with categories researched. In the second stage, the presentation of the data in this study used narrative text, tables, and pictures. In the third stage, which is the conclusion, the researchers interpreted the data that has been presented to answer research questions.

To ensure the findings, we applied within-method triangulation. We designed and implemented two different tasks for each category of understanding to see whether the findings converged to the same meaning. The research design was set as shown in Figure 1.

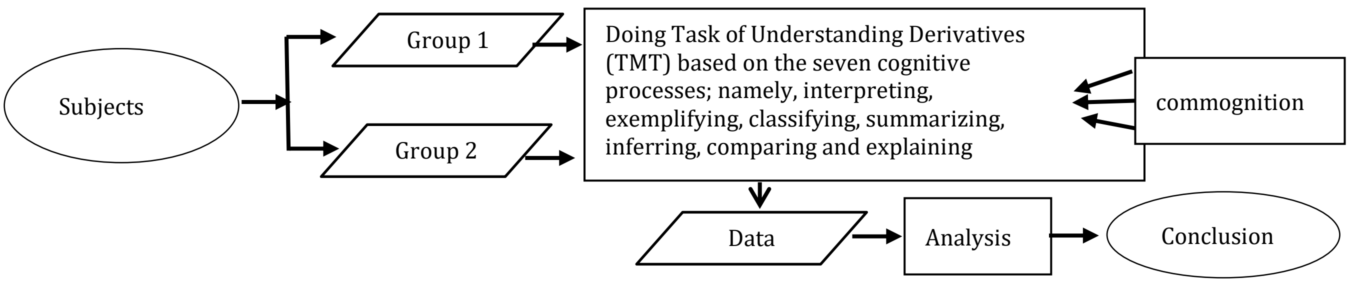


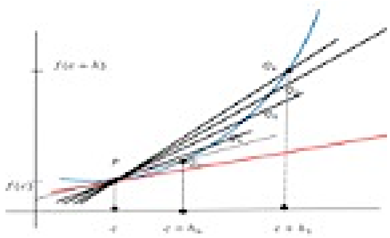
Figure 1. Research design

Findings

Interpreting Category

In the category of interpreting, the commognitive aspects that appear are keywords, visual mediators and endorsed narratives. Two problems TMT 1 and TMT 2 can be seen in Table 1.

Tabel 1. Problem TMT 1 and TMT 2 in the interpreting category

No	TMT 1	TMT 2
1.	Look at the image below  $P, Q_1, Q_2, \cdots Q_n$ are the intersection points of the secant lines and the curve. Express the definition of the derivative f at $x = c$ based on the above information in your own words.	Given the following derivative formula $(i).\, f'(c) = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$ $(ii).\, f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ Express the derivative definitions based on the above formulas in your own words!

From TMT 1, the subjects in Group 1 (RIFD, FEFI, HAFI) and in Group 2 (MIFI, KAFD, PUFD) were asked to define the derivatives by changing the form of representation from images to words in the interpreting category. Keyword aspect that appeared in this category were all subjects could say "curve," "tangent line," and "secant line." All subjects also mentioned the color of each element that they called.

Visual mediator aspect can be seen when the subject raises a symbol based on the given image. The point Q_1, the intersection of the secant line with the curve, can be symbolized by FEFI as $(c, f(c))$ and $\left(c + h_1, f(c + h_1)\right)$. They gave the symbol h_1 to represent the difference between abscissas of P and Q_1, and drew a mark on the task sheet. Furthermore, MIFI said that the slope of the PQ_1 line was $f(c + h_1) - f(c)$ per h_1, as well as for the line slope of $PQ_2, PQ_3, \cdots, PQ_n$ and symbolized the letter m for the slope.

Furthermore, the subject was directed by saying if $n \to \infty$ and $Q_n \to P$. The obtained result was that PUFD responds by saying h_n tends to zero and writing it down as $h_n \to 0$. Moreover, RIFD said limit h goes to zero. This means that subjects already use endorsed narratives. All subjects have been able to mention the definition of derivative by using some endorsed narratives. According to HAFI, the derivative is a function $y = f'(x)$. The graph of the original function has a tangent line at point P with a certain slope.

In TMT 2, subjects were asked to change the form of representation from the derivative formula to the form of words in providing derivative definitions. Here, more of the visual mediator aspects were apparent. HAFI read symbol $f'(c)$ as "f accent c," FEFI and RIFD read "derivative of function f (c)" which denotes the first derivative of f in c. MIFI from Group 2 read the symbol $f'(c)$ with "f accent c," PUFD read "derivative of $f(c)$" and "derivative of function $f(c)$." The reading of the symbol $f'(c)$ almost close to the endorsed one but they were more familiar with the symbol $f'(c)$ pronounced as "derivative of f in c" or "derivative of $f(c)$."

Furthermore, the symbol $\lim\limits_{h\to 0}$ is pronounced as "limit h goes to zero," "h approaches zero" and the symbol $\lim\limits_{x\to c}$ is pronounced as "x approaches c," "x goes to c." The HAFI pronounces the symbol $\frac{f(c+h)-f(c)}{h}$ by saying "the difference between the function $f(c+h)$ minus $f(c)$ and h," and FEFI read "the difference between $f(c+h)$ and $f(c)$ per h." The symbol $\frac{f(x)-f(c)}{x-c}$ is read by RIFD as "the difference between the quotient $f(x)$ and $f(c)$ with $x - c$" and HAFI read "the difference between the quotient of $f(x)$ and $f(c)$ and $x - c$."

While MIFI from Group 2 pronounced the formula $\lim\limits_{h\to 0}\frac{f(c+h)-f(c)}{h}$ with "the limit value h to zero which is the quotient of $f(c+h) - f\ (c)$ per h, " KAFD read " limit quotient of $f(c+h) - f\ (c)$ per h with h approaches zero," and PUFD read "limit quotient of function $f(c+h) - f\ (c)$ per h where h goes to zero or approaches zero." While the formula $\lim\limits_{x\to c}\frac{f(x)-f(c)}{x-c}$ is pronounced by PUFD as "limit is the quotient of $f\ (x) - f\ (c)$ per $x - c$ where x tends to zero," and KAFD as "limit $f\ (x)$ minus function $f\ (c)$,... *silent and continue*.... quotation of function $f\ (x)$ minus function $f\ (c)$ per $x - c$ where x approaches c," "limit quotient of $f\ (x) - f\ (c)$ per $x - c$ as x approaches c."

The difference in the pronunciation of the symbol will be more appropriate to read as "limit of $f(x)$ for h approaches 0" or "limit as h approaches 0 of $f(x)$ or "limit function $f(x)$ for x approaches 0. " All subjects name the symbols $\frac{f(c+h)-f(c)}{h}$ and symbols $\frac{f(x)-f(c)}{x-c}$ based on the form of the function that they see. There is no subject who reads by saying "the quotient of the difference between $f(c+h) - f(c)$ and h." They have not been able to synchronize the symbols and how to read the symbols.

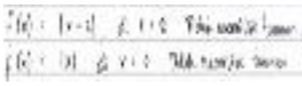
Flexibility on category exemplifying

In the exemplifying category, the commognitive aspect that appears is the flexibility of ritual routine. In TMT1, the subjects are asked to name two examples of functions that have derivatives. In TMT 2, two examples of functions that have no derivatives. The functions mentioned by subjects are as shown in Table 2.

Table 2. Transcript of discussion in exemplifying category

Group 1		**Group 2**	
TMT 1		**TMT 1**	
Interview	Discussion Transcript	Interview	Discussion Transcript
Researcher	Please read the problem *Subjects discussing*.	Researcher	Give two examples of functions that have a derivative at one point
RIFD	We mention the function?		*Wrote*
Researcher	Yes, the function is mentioned	MIFI	$f'(4) = 13x - 6$
FEFI	oh ya...$f(x) = x - 1$		
Researcher	At point?		
HAFI	at $x = 1$	KAFD	*Wrote* $f(x) = x^2 - 2 \quad x = 2$
Researcher	Have a derivative at that point?		
HAFI	Yes, one		
RIFD	One	PUFD	*Wrote*
Researcher	another sample?		$f(x) = 2x - 4$ dititik $x = 2$
HAFI	$f(x)$ is equal to x to the power of 2 where x is zero	Researcher	Are you sure the function has a derivative? subject: we shall discuss first
Researcher	Is it permissible at the point $x = 3$?		
RIFD	can be	KAFD	the function has a derivative, (*pointing to Mirna's paper*) 2) $f(x) = x^2 - 2 \quad x = 2$ $f'(c) = \lim\limits_{h\to 0}\frac{f(c+h)-f(c)}{h}$ $f'(2) = \lim\limits_{h\to 0}\frac{f(2+h)-f(2)}{h}$ $f'(2) = \lim\limits_{h\to 0}\frac{(2+h)^2-2-(2)^2-2}{h}$ $= \lim\limits_{h\to 0}\frac{((4+4h+h^2)-2)-((4)-2)}{h}$ $= \lim\limits_{h\to 0}\frac{4+4h+h^2-2-4+2}{h}$ $= \lim\limits_{h\to 0}\frac{4h+h^2}{h}$ $= \lim\limits_{h\to 0} 4+h = 4$ *They tested using the formula in question 1 part b*
HAFI	at point $x = 4$?		
RIFD	$f(x) = x - 1$, at $x = 1$		
HAFI	$f(x) = x^2$, at $x = 0$		
Researcher	Oke		

Table 2. Continued

Group 1		Group 2	
TMT2		**TMT 2**	
Interview	Discussion Transcript	Interview	Discussion Transcript
Researcher	What are two examples of functions that have no derivatives?	Researcher	Give two examples of functions that have no derivative at one point
RIFD	there is		*Subjects looked at each other*
Researcher	What functions?	MIFI	The function's absolute value x at $x = 0$
RIFD	$f(x) = 0$ and $f(x) = x$	KAFD	The function does not have a derivative, if we look for it using a derivative formula.
Researcher	Does it have no derivatives?	KAFD	$f(x) = \|x\|$, $x = 0$ has no derivative
FEFI	Yes, derivative one	MIFI	*Mirna wrote the following* 
FEFI	for example a constant function $f(x)$ equals to 5		
Researcher	Does it have no derivatives?		
RIFD	there is		
Researcher	Its value?		
RIFD	Zero		
HAFI	Functions that take absolute values		
HAFI	$f(x)$ equals absolute value of x at $x = 0$ (*They replied almost together*)		
Researcher	has no derivatives? Are Sure		
RIFD	sure		
Researcher	Ok, one more		
RIFD	Absolute of function $x - 2$ at $x = 2$		
FEFI	*Wrote down what his friend has mentioned* 		
Researcher	Can you explain?	Researcher	another example?
	I cannot explain (*said the subject in a slightly soft voice*)		Not yet, Mom (*subject answered*)

All subjects of Group 1 responded well to this question by providing two examples of the requested function. The function mentioned by FEFI is $f(x) = x - 1$ at $x = 1$ and by HAFI is $f(x) = x^2$ at $x = 0$. The function given by the subject is correct, but she repeated the function written in the previous question. In TMT 2, the subjects were asked to give two examples of functions that have no derivatives. At first, RIFD seemed confused to answer because she mentioned two functions, namely $f(x) = 0$ and $f(x) = x$. She was redirected by asking, "does the function that you wrote have no derivatives?" HAFI said that "functions that have absolute value have not derivatives." The word "have," which is meant by her has an absolute value. She then corrected her answer by writing the function $f(x) = |x - 2|$ at $x = 2$ and $f(x) = |x|$ at $x = 0$.

All subjects in Group 2 mentioned three examples of functions that have derivatives, namely $f'(4) = 13x - 6$, $f(x) = x^2 - 2$ at $x = 2$ and $f(x) = 2x - 4$ at $x = 2$. In TMT 2, the subjects could only provide one of the two requested examples for a function that has no derivative. The function written by KAFD was $f(x) = |x|$ at $x = 0$.

Based on TMT1 and TMT 2, the functions presented by the subjects do not vary because they are limited to simple functions and retrieve the existing ones. The subjects should be able to provide a variety of examples so that it is not stiff. In this category the examples given by the subjects are correct, but these are very rigid or there are no differences in providing the examples requested. To these two questions, the subjects could provide examples that are required by the problems, however the examples given by the subject do not vary.

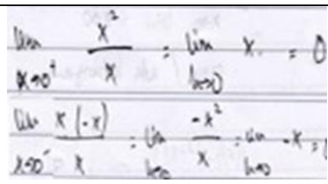
By whom the routine is performed on classifying category

In the classifying category, the subjects could classify functions that have derivatives and functions that have no derivatives. However, they cannot explain functions that have derivative or do not derivative. Table 3 shows a snippet of the subjects' discussion.

Table 3. Transcript of discussion in classifying category

Group 1		**Group 2**	
TMT 1		**TMT 1**	
Interview	Discussion Transcript	Interview	Discussion Transcript
Researcher	Consider question number 3		*The subject begins to discuss which function has a derivative*
FEFI	question number (a) is answered, has no derivative	PUFD	Problem a) steps function and has no derivatives (*the subject answers almost simultaneously)*
Researcher	Problem b, what the function is?		
FEFI	step function		
	ok steps function? (*the other two subjects looked confused*)	KAFD	Confused with the sign on the question sheet b, but familiar with the notes beside the question.
Researcher	How about writing $[\![x]\!]$? (*write on the blackboard*)		
RIFD	ooo..It is a step function mam	Researcher	write other symbols on the board $[\![x]\!]$
FEFI	I thought it was the same as question number a)	KAFD	it is a steps function Steps function has no derivatives (they answer simultaneously)
HAFI	The largest integer that is less than or equal with x	Researcher	What is the reason?
Researcher	does this function have any derivatives?		We cannot explain Mom yet (*she looks like shy)*
RIFD	Has no derivatives, ma'am		they discuss and write on their respective papers
	Try to explain why it doesn't have a derivative?		Problem c
	For a while the subject was silent then they answered, we cannot	PUFD	*Wrote* $F(x) = \|x-1\|$ di $x=1$ $\quad \|x-1\| = \begin{cases} x-1 & \geq 1 \\ -x+1 & \leq 1 \end{cases}$
Researcher	How about c?	MIFI	*Wrote* $\lim \frac{\|x-1\|-1}{x-1}$ $\lim_{x \to 0^+} \frac{x-1-1}{x-1} = \lim_{x \to 0^+} \frac{x-2}{x-1} = 2$ $\lim_{x \to 0^-} \frac{-(x-1)-1}{x-1} = \lim_{x \to 0^-} \frac{-x}{x-1} = \frac{0}{0-1} = 0$
HAFI	Has no derivatives (simultaneously)		
		Researcher	Why is the limit x going to 0? (*Subject says the function in question c has no derivative.*)
TMT 2		**TMT 2**	
Interview	Discussion Transcript	Interview	Discussion Transcript
Researcher	Consider question number 3	Researcher	Read out the questions (*Subject is listening*)
HAFI	Problem d) has a derivative		
HAFI	e) also has a derivative	KAFD	d has a derivative, because it is similar to the example we gave in problem 2. (*followed by other subjects*)
Researcher	be careful, try to have an attention again		
FEFI	this is x times $\|x\|$		Problem e). (*They began to hesitate again to answer it.*)
Researcher	Try to write down the meaning of $\|x\|$		
HAFI	x for x is equal to or equal to 0 and negative x for x is less than zero.	KAFD	Use absolute function again?
		MIFI	problem e) has no derivative
	Wrote $\|x\| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ $f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x-c}$	Researcher	Recall the definition of absolute value
		MIFI	We try to write first, (*They write on their own papers.*)
			Recall the first derivative formula and the definition of absolute value
Researcher	Use the formula you wrote down.		
FEFI	$f'(x) = \lim_{x \to c} \frac{x\|x\|-0}{x-0}$ (*Furthermore, the subject is asked to do*)	KAFD	(*The results of their discussion were written by Kamila.*)
HAFI	$f'(x) = \lim_{x \to 0} \|x\|$		$f'(0) = \lim_{h \to 0} \frac{x\|x\|-0}{x-0}$ $= \lim_{h \to 0} \frac{x\|x\|}{x}$

Table 3. Continued

Group 1		Group 2	
TMT 2		**TMT 2**	
FEFI	x is approximated to the right of its absolute value, positive		
FEFI	x is approximated to the left of its absolute value, negative		
FEFI	the value is zero, has a derivative (*followed by two other subjects*)		
Researcher	does problem f have a derivative?	PUFD	It means that the question e) has the derivative
FEFI	yes it has the derivative at $x = 0$		
RIFD	has derivatives d, e and f.	MIFI	I think everyone that has an absolute sign does not have a derivative
		Researcher	you cannot conclude that
			Problem f) has a derivative (the subject answers simultaneously)
			So the problem has the derivatives d, e and f.

All subjects from Group 1 said the functions $f(x) = |x|$, at $x = 0$, $f(x) = [\![x]\!]$, at $x = 0$, and $f(x) = |x - 1|$, at $x = 1$ are functions that have no derivatives. Furthermore, the subjects said that both functions $f(x) = x^2$ and $f(x) = x^2 - 1$ have derivative at $x = 0$. When the subjects consider the function $f(x) = x|x|$, at $x = 0$, they start to look doubtful because their answers are different. Some answer that they have a derivative or have no derivative. FEFI responded by saying "this is the multiplication of x and $|x|$." They were asked to name the definition of $|x|$. HAFI said "x for x is greater or equal to zero and negative x for x is less than zero". She could define the absolute value. Furthermore, she wrote the first derivative formula by

$f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$. The next process, she was directed to use the derivation formula by replacing $f(x) = x|x|$ and $f(c = 0) = 0$.

$f'(x) = \lim_{x \to 0} \frac{x|x| - f(c)}{x - c} = \lim_{x \to 0} \frac{x|x| - 0}{x - 0} = \lim_{x \to 0} |x|$

FEFI defined the latter function by saying "if x is approached from the right, then the x value is positive" and "if x is approached from the left, the x value is negative." As a result, she concluded that the function has a derivative at $x = 0$. Based on the several functions presented, the subjects can be said to have not fully understood the concept of derivative functions. This is because they cannot explain the reasons for the answers they provide and their answer process needs to be scaffolded from other people.

All subjects of Group 2 said that $f(x) = |x|$ and $f(x) = [\![x]\!]$are functions that have no derivative at $x = 0$. They gave a correct answer, but they could not explain it. They said that the function $f(x) = |x - 1|$ has no derivative at $x = 1$, but also with no explanation. They were directed by recalling the nature of the absolute value that they have written with an endorsed narrative and related it to the first derivative formula. MIFI wrote down $f'(1) = \lim_{x \to 0} \frac{|x-1|-1}{x-1}$.

Furthermore, she continued the process by writing the right limit and left limit as follows: $\lim_{x \to 0^+} \frac{x-1-1}{x-1} = 2$, $\lim_{x \to 0^+} \frac{-(x-1)-1}{x-1} = 0$.

She concluded that the function has no derivative because the right limit value is not the same as the left limit value. There is something interesting here. First, the subjects saw that the value of the left limit is different from the right limit, then they concluded that the function has no derivative. Second, due to the symbols used, they did not pay attention to the problem at which point the function has no derivative.

In TMT 2, for the functions $f(x) = x^2$, at $x = 0$ and $f(x) = x^2 - 1$, at $x = 0$, they said that they were already familiar with. In the function $f(x) = x|x|$ at $x = 0$ they began hesitant to answer. Subjects were directed to recall the formula for the first derivative and the definition of absolute value that they had written previously. KAFD used the first derivative formula, namely

$$f'(1) = \lim_{x \to 0} \frac{f(x) - f(1)}{x - 0} = \lim_{x \to 0} \frac{x|x| - 0}{x - 0} = \lim_{x \to o} \frac{x|x|}{x}$$

Furthermore, she completed the function above by determining the value

$$\lim_{x \to 0^+} \frac{x^2}{x} = \lim_{x \to 0^+} x = 0 \text{ and } \lim_{x \to 0^-} \frac{-x^2}{x} \lim_{x \to 0^-} (-x) = 0.$$

Based on the results obtained, they said that $f(x) = x|x|$ has the derivative at $x = 0$ whether the terms used to refer to the right limit, left limit, right derivative or left derivative. It is better if this is clear in order to differentiate from the definition of continuity. Although in the end they can make correct conclusions from the information provided. This category is also included in the ritual routine "by whom the routine is performed."

By whom the routine is performed on summarizing the category

In the summarizing category, TMT 1 and TMT 2 were given by several functions with formula information and their properties, the subjects were asked to determine "What are the characteristics of a function that has a derivative?"

The subjects can say the keywords "continuous function" and "limit function" based on the information provided in the questions. This is similar to TMT 2, the subjects only mentioned "continuous function" and "discontinuous function." Additional keywords from them in Group 2 are "left limit" and "right limit."

HAFI said that the characteristic of a continuous function is if the function limit exists. She could not explain further, stayed mostly silent. She did not attempt to assess the discontinuous function even though the information was contained in the TMT. The question on TMT was read again, FEFI said, "the characteristic of a function that has a derivative is if the limit value exists."

Furthermore, all subjects in Group 2 divided the functions into three groups based on the formula information and the properties provided in the problem, namely "continuous functions have no derivatives," said by KAFD. In contrast, MIFI said, "discontinuous functions have no derivatives, and continuous functions have derivatives." KAFD said that a continuous function means that its left limit and right limit are equal. MIFI added by saying that a function is discontinuous, meaning that its left and right limits are not equal. Furthermore, KAFD said that a function has a derivative if its limit exists.

Based on the responses given by the subjects, it appears that the formula information and the properties given in the questions have not been completely elaborated. They have not yet fully said the condition for a continuous function. On the other hand, they also have not distinguished between continuous functions and features of functions that have derivatives. In this case, they can be said to have not fully understood the concept of derivative functions and need to be scaffolded from others.

Applicability on Inferring category

In the inferring category, TMT 1 problem is as follows: Given the formula $f(x) = \begin{cases} \frac{1}{2}x^2 \ , \ x < 3 \\ 3x \ , \ x \geq 3 \end{cases}$, the function f has a derivative at $x = 3$, with the value $f(3) = 9$ and f is continuous at $x = 3$. Based on the information above, what can you conclude? In TMT 2 problem, the function $f: [0,2] \to R$ which is defined as $f(x) = \begin{cases} 2x, \ 0 \leq x < 1 \\ 1, \ 1 \leq x \leq 2 \end{cases}$ is not continuous at $x = 1$, f has no derivative at $x = 1$. Based on the information above, what can you conclude?

All subjects in Group 1 looked like discussing. FEFI said based on the problem, the function f has a derivative at $x = 3$, then RIFD added by saying f is continuous at point $x = 3$. She said that f has a derivative at $x = 3$, the function value equals 9, which means that the function is continuous at that point. Then RIFD mentioned again that f is the function that has a derivative at that point, and it is continuous at that point. She clarified his answer by saying, "if f accents x in c has a derivative then it is continuous at $x = c$." As a result, he can conclude that if f has a derivative at x then f is continuous at x.

At TMT 2, HAFI of Group 1 quickly said, "if the function f has no derivative at point x then it is not continuous at point x." RIFD interrupted by saying, "it doesn't seem to work." She began to look doubtful with the answer he gave. The subject was directed back to pay attention to the information on the question. RIFD answered by saying, "if the function f is not continuous in x then it has no derivative in x."

Based on the information provided at TMT 1 question, KAFD in Group 2 said, "the function has derivative at some point if it is continuous at that point" (looks doubtful afterward). PUFD said, "if it is continuous, it means the left limit and the right limit are the same; it means the limit exists." Another statement from KAFD "if the left and right limits are the same, it means to have a derivative." They have conveyed different interpretations of the information from the questions. They were asked to pay more attention to the information in question because their responses had to be corrected. KAFD said, "a function has a derivative means that it is continuous at $x = 3$," added by MIFI, "The function has a derivative at $x = 3$ means it is continuous at $x = 3$ too." Furthermore, they said that if f has a derivative at c, then f is continuous at c.

The subjects can make correct conclusions based on the information provided. But there is some confusion about the definition in group, between a continuous function and a function that has a derivative. This category is also included in the "applicability" ritual.

Visual Mediator Aspect in comparing category

In the comparing category, from the TMT 1: What is the relationship between the derivative of the sum of two functions and the sum of derivatives of each function at the same point?

RIFD in Group 1 responded by saying, “the relationship is the same,” by writing two functions, $f_1(x) = x - 1$ and $f_2(x) = x^2$. The two functions that derived by students became $f_1{}'(x) = 1$ and $f_2{}'(x) = 2x$. The relationship meant by RIFD is $f_1{}'(x) + f_2{}'(x) = 1 + 2x$. HAFI also represented other forms that they were working with. She rewrote two functions, namely $f(x) = x^3$ dan$g(x) = x$, then stated$(f + g)'(x) = f'(x) + g'(x) = 3x^2 + 1$. They symbolized the relationship in question by $(f + g)'(x) = f'(x) + g'(x)$, and they said that both sides have derivatives. Based on their interpretation of the fragment, “sum of derivatives per function” and “the sum of derivatives of each function” They symbolized “$(f + g)'(x)$” and “$f'(x) + g'(x)$” as $(f + g)'(x) = f'(x) + g'(x)$. They have replaced the word “and” in the problem with “equal to.” Furthermore, they answered the question of the relationship between the derivative of the sum of two functions and the sum of derivatives of each function at the same point if “both of them have derivative.”

MIFI from Group 2 wrote the symbol $f(x) + g(x) = (f + g)(x)$ and said “two functions are added then derived?” She was then asked with the question, “what is the derivative of the function meant?” PUFD said the derivative of the sum of two functions (while pointing at the one he wrote) equals to the sum of the derivatives of two functions. In the question fragment of TMT 1, “the derivative of two functions and the sum of derivatives of each function,” she replaced the word “and” into the word “equal to.” PUFD then expressed it in the form of $(f + g)'(x) = f'(x) + g'(x)$. This form is precisely the definition of the sum rule in derivative material.

Furthermore, the question in TMT 2 given to the subjects is “What is the similarity of the sum limit and the sum of two functions at one point?” This question was answered by the HAFI by saying, “limit properties.” Her statement was responded again by asking, “what is the nature of the limit?” RIFD answered “maybe both of them have derivative.” In this case, she seemed hesitant to answer because they used the word “maybe.” This showed that they could only write down the limit properties by

$$\lim_{x\to c}[f(x) + g(x) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)\,.$$

The piece of question “derivative of the sum of two functions at one point” was ignored by all subjects, but they answered the TMT 2 question by saying “both have derivatives.”

Similar to Group 2, the subjects said “limit of a sum.” Their answers are inadequate because there are no equalities that can be seen from what they mean.

In this category, the subjects only used a visual mediator and mentioned the endorsed narrative in the definition of limits, not finding the endorsed narrative during the problem solving process. However, the emerging endorsed narratives have not fully answered TMT 2 because it asks for the equality between the sum of limits and the derivative of the sum of two functions at one point. The subject only focuses on the “limit of a sum,” while the derivative of the number of functions is not explained. The subject only answered, “may be both have derivatives.” The correlation of finding the commognitive aspects between the keyword aspect and the visual mediator in the discourse raises an endorsed narrative aspect. This is in line with the idea of Sfard (2008) and Robert and Roux (2018).

Explaining Category

In TMT 1, assuming that f is an odd function, the subjects use the definition of the first derivative to relate the derivative of an odd function to an even function. TMT 2 problem: A function f has the symmetric derivative f_s' which is defined by $f_s'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x-h)}{2h}$ exists. Explain that $f_s'(x) = f'(x)$. Table 4 shows the explanation from the subjects.

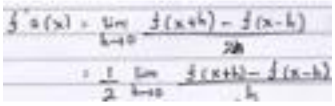
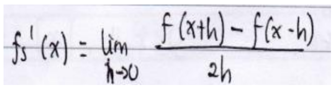
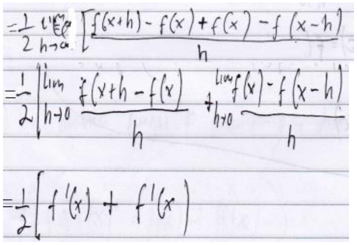
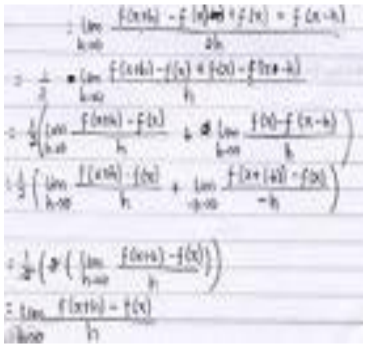
Table 4. Transcripts of discussion in the Explaining category

Group 1		Group 2	
TMT1		**TMT 1**	
Interview	Discussion Transcript	Interview	Discussion Transcript
Researcher	Please read question number 7	Researcher	What are you assuming?
	Subjects read question	KAFD	odd or even function huh?
FEFI	Assume an odd function		make sure, the function is odd
FEFI	For example, if $f(x)$ is replaced by $-x$, the final result is $-f(x)$.	Researcher	or even?
	Wrote		Like this?
FEFI	$f(x) = -f(x)$	MIFI	$f(x) = -f(x) \longrightarrow$ fungsi ganjil ~~$f(x) = f(x)$~~
HAFI	Odd function		$f(x) = -f(x) \rightarrow$ odd function
HAFI	f negative x equals to negative $f(x)$	KAFD	yes like this (*pointing to what Mirna wrote*)
Researcher	Try to write it down correctly!		

Table 4. Continued

Group 1		**Group 2**	
TMT 1		**TMT 1**	
HAFI	$f(-x) = -f(x) \rightarrow$ fungsi ganjil	KAFD	I forgot, even though I had already studied
		PUFD	If f is negative x equals to negative $f(x)$ then f is an odd function
FEFI	derivative of odd function is an even function (*she repeated the problem*)	Researcher	What about even function?
Researcher	Write down the definition of the first derivative	KAFD	if f is negative x equals to $f\ (x)$ is an even function
RIFD	*Wrote* $f'(x) = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$	MIFI	the derivative of odd function is the even function (*repeats reading the problem*)
Researcher	Assume what problem?	Researcher	look again to at the question, what you have to do
HAFI	f is an odd function, which means that x is replaced by $-x$		*The subject looks confused*
FEFI	*Wrote* $f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h}$ $f'(-x) = \lim_{h \to 0} \frac{f(-x+h) + f(x)}{h}$ $= \lim_{h \to 0} \frac{f(-x+h) + f(x)}{h}$ $= \lim_{h \to 0} \frac{f(-(x-h)) + f(x)}{h}$	Researcher	Do you remember the definition of the first derivative?
		PUFD	Can I use c? *wrote* $f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ $f'(-c) = \lim_{h \to 0} \frac{f(-c+h) - f(-c)}{h}$
		Researcher	Yes, please continue
		MIFI	*Wrote* $f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h}$ $= \lim_{h \to 0} \frac{f(-(x-h)) - f(-x)}{h}$
Researcher	Why does it become + f (x)?		
HAFI	because of an odd function	KAFD	*Writing almost similar to Mirna*
Researcher	what property do you use (*pointing to* $-\ f(x-h)$*)* odd function nature (*the subject answers almost simultaneously*)	PUFD	*pay attention briefly while writing on the paper.*
FEFI	*Feni continues his work*		*Subject looks confused (they look at each other)*
RIFD	*others pay attention)* $f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h}$ $= \lim_{h \to 0} \frac{f(-x+h) + f(x)}{h}$ $= \lim_{h \to 0} \frac{f(-(x-h)) + f(x)}{h}$ $= \lim_{h \to 0} \frac{-f(x-h) + f(x)}{h}$ $= -\lim_{h \to 0} \frac{f(x-h) + f(x)}{h}$ $= -\lim_{h \to 0} \frac{f(x+(-h)) + f(x)}{h}$ $= -\lim_{-h \to 0} \frac{f(x+(-h)) + f(x)}{-h}$ $= -(f'(x))$ $= f'(x)$	KAFD	*look* $f'(-x) = \lim_{h \to 0} \frac{-(f(x-h) - f(x))}{h}$ $= \lim_{h \to 0} \frac{f(x+(-h)) - f(x)}{-h}$ $f'(-x) = -(-f'(x))$ $f'(-x) = f'(x)$
		Researcher	Can you explain that there is -*h*?
		KAFD	Not yet
		Researcher	is every odd derivative an even function?
RIFD	So the derivative of this odd function is an even function *make an example* Contoh $f(x) = x^5$ $f'(x) = 5x^4$	MIFI	Yes. Look that $f(x) = x^3 \rightarrow$ fungsi ganjil $f'(x) = 3x^2 \rightarrow$ fungsi genap $f(x) = x^3 \rightarrow$ odd function $f(x) = 3x^2 \rightarrow$ oven function
Researcher	ok	PUFD	Yes, mam. $f(x) = x^2, f'(x) = 2x$ *Mifi gave more ideas. his friends just carry on*

Table 4. Continued

Group 1		Group 2	
TMT 2		**TMT 2**	
Interview	Discussion Transcript	Interview	Discussion Transcript
Researcher	Please read the questions and think	Researcher	Consider question number 7. Have you studied the derivative of symmetric functions?
HAFI	Starting from the definition of a symmetric function *Wrote*	PUFD	not yet
FEFI		Researcher	Consider a symmetric function like the definition in the problem *The subject writes a symmetrical derivative function*
Researcher	OK. what should you do again? *The subject discussed while playing the pen*	MIFI	*Wrote* 
HAFI	*write down the results of their discussion* 	Researcher	Ok. Explain that $f_s'(x) = f'(x)$
		PUFD	*Seems to prefer to work alone*
FEFI	The right portion marked equals to f accents x $\lim\limits_{h\to 0}\frac{f(x)-f(x-h)}{h} = f'(x)$	MIFI	Look that 
Researcher	Would you please explain the reason?		
HAFI	Don't know Mom (*they kind of smile*) So, $f_s'(x) = f'(x)$	KAFD	can be $f_s'(x) = f'(x)$
		Researcher	ok. now the explanation of each step includes $\lim\limits_{-h\to 0}\frac{f(x+(-h))-f(x)}{-h}$?
		MIFI	Not yet

In TMT 1, assuming that f is an odd function, the subject uses the definition of the first derivative to relate the derivative of an odd function to an even function. HAFI could symbolize $f(-x) = -f(x)$ as an odd function and $f(-x) = f(x)$ for an even function.

At each step, the subjects have noticed the use of odd function assumptions. RIFD started to write the first derivative formula as follow: $f'(c) = \lim\limits_{h\to 0}\frac{f(c+h)-f(c)}{h}$. Furthermore, assuming $f(-x) = -f(x)$, FEFI wrote:

$$f'(-x) = \lim_{h\to 0}\frac{f(-x+h)-f(-x)}{h} = \lim_{h\to 0}\frac{-(f(x-h)-f(x))}{h} = -\lim_{h\to 0}\frac{f(x+(-h))-f(x))}{-h}$$

$$f'(-x) = -(-f'(x))$$

$$f'(-x) = f'(x)$$

RIFD displayed the symbol $-h$, but she could not explain it clearly. The process carried out by her is correct while still being given guidance and direction. However, she cannot explain the specific steps that have been carried out.

Furthermore, in TMT 2, FEFI only rewrote the symmetric function symbol contained in the problem by:

$$f_s'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x-h)}{2h} = \frac{1}{2}\lim_{h\to 0}\frac{f(x+h)-f(x-h)}{h}.$$

The HAFI wrote down the symbol

$$f_s'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)+f(x)-f(x-h)}{2h}$$
$$= \frac{1}{2}\left[\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + \lim_{h\to 0}\frac{f(x)-f(x-h)}{h}\right]$$
$$= \frac{1}{2}[f'(x)+f'(x)]$$
$$f_s'(x) = f'(x)$$

At the end of the process, FEFI raises a symbol $\lim_{h\to 0}\frac{f(x)-f(x-h)}{h} = f'(x)$. Subjects were asked to describe the symbols they wrote. Unfortunately, they could not explain it.
Furthermore, TMT 1 in Group 2, the subjects have symbolized the even function and odd function in the visual mediator aspect. In the next step, they looked confused because they read the questions over and over and in whispers. They were reminded to use the definition of the first derivative to relate with the definition of an odd function. KAFD started to write the first derivative formula as follows:

$$f'(c) = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

Assuming $f(-x) = -f(x)$, MIFI writes: $f'(-x) = \lim_{h\to 0}\frac{f(-x+h)-f(-x)}{h}$

$$f'(-x) = \lim_{h\to 0}\frac{-(f(x-h)-f(x))}{h} = -\lim_{h\to 0}\frac{f(x+(-h))-f(x))}{-h} = -(-f'(x)) = f'(x)$$

KAFD displayed the symbol -*h,* but she could not explain clearly. The process that she carried out is correct while still being given guidance and direction. However, she cannot define the specific steps she carried out.

In TMT 2, MIFI started with the known definition of the symmetric function and the definition of the first derivative

$$f_s'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x-h)}{2h}$$
$$= \lim_{h\to 0}\frac{f(x+h)-f(x)+f(x)-f(x-h)}{2h}$$
$$= \frac{1}{2}\left(\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + \lim_{h\to 0}\frac{f(x)-f(x-h)}{h}\right)$$
$$= \frac{1}{2}\left(\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + \lim_{-h\to 0}\frac{f(x+(-h\_)-f(x)}{-h}\right)$$
$$= \frac{1}{2}\left(2\left(\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\right)\right) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x)$$

She can solve this problem, but she did not provide the reasons why she can say that

$$\lim_{-h\to 0}\frac{f(x+(-h)-f(x)}{-h} = f'(x).$$

In both TMT 1 and TMT 2 tasks, in this case, she can be said to be included in the category of ritual "closing conditional" or can complete the procedure.

Discussion

This research focuses on the social aspect because learning is carried out in groups in completing derivative tasks. Students can exchange mathematical ideas with their group mates through their discourse when completing the assignment. Commognition is a discursive theory that is useful for describing the learning process. Collaborative activities carried out in this study are social endeavors and can encourage students to build knowledge that comes from outsiders.

The mathematics discourse that students acquire in completing derivative tasks is also a process of individualization. In the individualization process here, students gradually become discourse actors. The discourse aspects that emerged during the discussion were keywords, visual mediators, endorsed narratives, and routines. The following will be discussed based on the findings obtained.

Keywords which had been mentioned previously are possibly the smallest verbal component of the subject's speech. In this study, we focus on the keywords that are used by the students, namely mathematics keywords. The words spoken by the students were checked by paying attention to whether they replaced everyday words with mathematics words. They have been in the objective use phase because their keywords are already classified as nouns. The use of symbols and keywords that were employed by them is a feature of formal discourse. This means they have started an exploratory discourse (Robert & Roux, 2018; Sfard, 2008).

The visual mediator aspect, a visual representation of mathematical objects, in this case, the derivative objects, appears in the interpreting, classifying, comparing, and explaining categories. In these four categories, the visual mediator is in the form of algebraic symbols and notations, mostly used by the students. However, some students also used physical objects, such as index fingers to facilitate their communication. Objects such as symbols, graphs, algebraic formulas that correlate the relationships and operations with mathematical objects used in interviews are called visual mediators (David & Tomaz, 2012; Nardi et al., 2014; Sfard, 2008).

The objects such as symbols, graphs, algebraic formulas that connect the relationships and operations with mathematical objects used in interviews are called visual mediators (Sfard, 2008; David & Tomaz, 2012; Nardi et al.,2014). Visual mediators that appear in the three categories are visual representations of mathematical objects operated by the students. Algebraic symbols and notations are used mainly by them. The correlation between the keyword aspect and the visual mediator in the discourse raises an endorsed narrative aspect. This is in line with Sfard (2008) and Robert and Roux (2018) idea.

Routine aspects as part of formal mathematics, a different patterned way, partly emerged when the students discussed derivatives. The routine characteristics in this paper are flexibility, by whom the routine is performed, applicability, and closing conditional. This mathematical regularity can be seen in how they use keywords and visual mediators, thus acquiring new narratives or strengthening existing narratives. These findings only reinforce the existing narrative.

In these two questions at exemplifying category, the students could provide examples requested by the problems. However, the examples given by the subject do not vary. They should be able to provide a variety of examples so that it is not stiff. This includes the ritual "flexibility," which means that the examples are given only follow those that already exist. In this category, the examples given by the students are correct but very rigid, or there is no variation in providing the examples requested. This shows that they are still fixated on assumptions that the important thing can answer the questions and correct. The results in this category being included in the routine flexibility ritual indicate that the examples given are limited, following the idea of Sfard (2008), Thoma and Nardi (2018), and Robert and Roux (2018).

The analysis shows that the students in the discussion of solving the task of understanding derivatives have not been independent. They still follow directions from others and imitate the work of fellow discussion members. In line with Sfard (2020), they are not yet independent, so that they often follow other people's metarules and use 'thoughtful imitation.' However, in this study, the word imitation does not mean that they only imitate without thinking and communicating.

The applicability of the research findings is very narrow because it only applies to ongoing discussions. After all, the conclusions obtained cannot be generalized. This happened as in the summarizing category.

Routines generally occur in situations that are often experienced in the past and repeatedly happen. To explain what happened is to say that the students' action was an attempt to imitate something they had seen and done. Learning is the ability to react to new situations by making use of the memories that have been acquired. The tendency to model our current actions on what has been done in the past produces patterns of action that we call routines. As a result, it can be said that routine activity is the essence of learning. Furthermore, routine is the most basic thing in which all the creativity of the students is rooted. This is a medium for them to express what they found (Lavie et al., 2019). In the category of exemplifying, classifying, summarizing, inferring, and explaining, a type of routine "ritual" was found through the individualization process in carrying out derivative tasks. The ritual routines obtained are process-oriented and include discursive routines because interpreting task situations requires communication actions following the idea of Lavie et al. (2019).

Conclusion

The results showed that not all students' discourse characteristics appear in every category of the seven cognitive processes. In group one, keywords from nouns and formal words occurred during interpreting, classifying, and summarizing. Visual mediators appear in interpreting, classifying, comparing, and explaining. Students use symbols and physical objects as communication media. The definition as a narrative characteristic of discourse appears during interpreting, classifying, and comparing. Routines appear only in rituals. In the routines category, rituals appear in exemplifying, classifying, summarizing, inferring, and explaining. In the exemplifying category, "flexibility" appears as a ritual routine. In the category of classifying and summarizing, there is a discourse on ritual routine "by whom the routine is performing." In the inferring category, "applicability" emerges as a ritual routine.

Meanwhile, in the explaining category, "closing conditional" emerges as a discourse on the ritual. Furthermore, in the second group, the cognitive aspects, namely keywords, visual mediators, and endorsed narratives, appear in the same category as group one. On the other side, in the routines aspect, there are some differences in the classification category.

In the second group, "applicability" emerges as a routine discourse on the ritual. Meanwhile, other forms of communication, namely gestures, appear in several categories of cognition. In Group 1, students with FI style were more active, analytical, and more able to explain. Students with FD style tended to add answers, repeat other students' answers, and provide conclusions. In Group 2, FI students were more active, analytical, and explained their answers well. FD students were more likely to write down what they think, then explain by reading what was written. The activities of each group occurred because of the individualizing process.

Recommendations

This study uses the commognition theory by Sfard (2008), which assumes the learning is a patterned collective activity. According to the main assumption of commognitive, thinking is defined as the activity of communicating with oneself. The ontological principle about mathematics and its learning claim that mathematical thinking can be seen as a discourse that refers to a particular type of communication. The term discourse applies to communication that distinguishes the characteristics, namely keywords, visual mediators, endorsed narratives, and routines. Furthermore, the important developing aspect in this study is how the subject learns the basics of derived material. For example, how to read the symbol appropriately and correctly. In completing the task of understanding derivatives, several spontaneous movements rise from the subjects. The movements cannot be separated from the process of thinking and communication. As a result, these need to be further investigated.

Limitations

The limitation of this study, namely the difficulty in exploring the subject's cognition and communication during a discussion, became a challenge. With different student cognitive styles, students' cognition in understanding derivatives will also be different. Therefore, further research needs to develop this kind of research.

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Authorship Contribution Statement

Lefrida: Conceptualization, design, drafting manuscript, writing and analysis. Siswono: Conceptualization, design, critical revision of manuscript, supervision. Lukito: Conceptualization, design, critical revision of manuscript, interpretation, supervision.

References

Anderson, W. L., & Krathwohl, D. R. (Eds.). (2001). *A taxonomy for learning, teaching, and assessing: A Revision of Bloom's taxonomy of educational objectives.* (Abridged ed.). Addison Wesley Longman.

Armstrong, S. J., Peretson, E. R., & Rayner, S. G. (2012). Understanding and defining cognitive style and learning style: A Delphi study in the context of educational Psychology. *Educational Studies*, *38*(4), 449–455. https://doi.org/10.1080/03055698.2011.643110

Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics. *The Journal of Mathematical Behavior*, *26*(4), 348–370. https://doi.org/10.1016/j.jmathb.2007.11.001

David, M. M., & Tomaz, V. S. (2012). The role of visual representations for structuring classroom mathematical activity. *Educational Studies in Mathematics*, *80*, 413–431. https://doi.org/10.1007/s10649-011-9358-6

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*, 103–131. https://doi.org/10.1007/s10649-006-0400-z

Hiebert, J., & Carpenter, T. P. (1996). *Learning and teaching with understanding.* National Reston, Virginia Council of Teachers of Mathematics.

Kepner, M. D., & Neimark, E. D. (1984). Test-retest reliability and differential patterns of score change on the Group Embedded Figures Test. *Journal of Personality and Social Psychology*, *46*(6), 1405-1431. https://doi.org/10.1037/0022-3514.46.6.1405

Lavie, I.,Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies Mathematics*, *101*, 53–176. https://doi.org/10.1007/s10649-018-9817-4

Lefrida, R., Siswono, T. Y. E., & Lukito, A. (2020). A commognitive study on field-dependent students' understanding of derivative. *Journal of Physics: Conference Series, 1747*, 1-7. https://doi.org/10.1088/1742-6596/1747/1/012025

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative Data Analysis*. Sage Publication, Inc.

Nachlieli, T., & Tabach, M. (2012). Growing mathematical objects in the classroom –The case of function. *International Journal of Educational Research*, *51-52*, 10–27. https://doi.org/10.1016/j.ijer.2011.12.007

Nardi, E., Ryve, A., Stadler, E., & Viirman, O. (2014). Commognitive analyses of the learning and teaching of mathematics at university level: The case of discursive shifts in the study of calculus. *Research in Mathematics Education*, *16*(2), 182–198. https://doi.org/10.1080/14794802.2014.918338

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. NCTM

Ng, O. (2016). The interplay between language, gestures, dragging and diagrams in bilingual learners' mathematical communications. *Educational Studies Mathematics*, *91*, 307–326. http://doi.org/10.1007/s10649-015-9652-9

Ng, O. (2018). Examining technology-mediated communication using a commognitive lens: the case of touchscreen-dragging in dynamic. *International Journal of Science and Mathematics Education*. *17*, 1173–1193. https://doi.org/10.1007/s10763-018-9910-2

Park, J. (2013). Is the derivative a function? If so, how do students talk about it? *International Journal of Mathematical Education in Science and Technology*, *44*(5), 624–640. https://doi.org/10.1080/0020739X.2013.795248.

Piaget, J. (1952). *The origins of intelligence of the child* (M. Cook, Trans.). W W Norton & Co. https://doi.org/10.1037/11494-000

Riding, R. J., Glass, A., & Douglas, G. (1993). Individual differences in thinking: Cognitive and neurophysiological perspectives. *Educational Psychology*, 13(3&4), 267-280. https://doi.org/10.1080/0144341930130305

Robert, A., & Roux, K. (2018). A commognitive perspective on Grade 8 and Grade 9 learner thinking about linear equations. Pythagoras - *Journal of the Association for Mathematics Education of South Africa*, *40*(1), 1-15. https://doi.org/10.4102/pythagoras.v40i1.438

Sfard, A. (2008). *Thinking as communicating: Human development, development of discourses, and mathematizing*. Cambridge University Press. https://doi.org/10.1017/CBO9780511499944

Sfard, A. (2020). Commognition. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 91-101). International Publishing Springer Nature. https://doi.org/10.1007/978-3-030-15789-0_100031

Tabach, M., & Nachlieli, T. (2016). Communicational perspectives on learning and teaching mathematics: Prologue. *Educational Studies in Mathematics*, *91*(3), 299–306. https://doi.org/10.1007/s10649-015-9638-7

Tallman, M. A., Carlson, M. P., Bressoud, D. M., & Pearson, M. (2016). A characterization of calculus I final exams in US colleges and universities. *International Journal of Research in Undergraduate Mathematics Education*, *2*(1), 105–133. https://doi.org/10.1007/s40753-015-0023-9.

Thoma, A., & Nardi, E. (2018). Transition from school to university mathematics: Manifestations of unresolved commognitive conflict in first year students' examination scripts. *International Journal of Research in Undergraduate Mathematics Education*, *4*, 161–180. https://doi.org/10.1007/s40753-017-0064-3

Varberg, D., Purcell, E. J., & Ridgon, S. (2010). *Kalkulus* (I. Nyoman, Trans.). Prentice Hall, Inc. (Original work published 2007)

Witkin, H. A., Oltman, P., Raskin, E., & Karp, S. (1971). *A manual for the embedded figures test*. Consulting Psychologists Press. https://doi.org/10.1037/t06471-000

Wittgenstein, L. (1953). *Philosophical investigations*: *The German text, with a revised English translation* (G.E.M. Anscombe, Trans). Blackwell Basil. (Original work published 1945)