The Interrelationships between Metacognition and Modeling Competency: The Moderating Role of the Academic Year

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Abstract: Several concerted movements toward mathematical modeling have been seen in the last decade, reflecting the growing global relationship between the role of mathematics in the context of modern science, technology and real life. The literature has mainly covered the theoretical basis of research questions in mathematical modeling and the use of effective research methods in the studies. Driven by the Realistic Mathematics Education (RME) theory and empirical evidence on metacognition and modeling competency, this research aimed at exploring the interrelationships between metacognition and mathematical modeling and academic year level as a moderator via the SEM approach. This study involved 538 students as participants. From this sample, 133 students (24.7%) were from the first academic year, 223 (41.4%) were from the second and 182 (33.8%) were from the third. A correlational research design was employed to answer the research question. Cluster random sampling was used to gather the sample. We employed structural equation modeling (SEM) to test the hypothesized moderation employing IBM SPSS Amos version 18. Our findings confirmed the direct correlation between metacognition and mathematical modeling was statistically significant. Academic year level as a partial moderator significantly moderates the interrelationships between the metacognitive strategies and mathematical modeling competency. The effect of metacognition on mathematical modeling competency was more pronounced in the year two group compared to the year one and three groups.

Keywords: Academic year levels, confirmatory factor analysis, mathematical modeling, metacognition, structural equation modelling.


Introduction

The last decade has witnessed several concerted movements toward mathematical modeling competency given the increasing worldwide relation of the role of mathematics in modern science, technology and real-life context. Modeling competencies engage the movement of knowledge and skills between contexts and is viewed as a basic feature of science, technology, engineering and mathematics (STEM) education (Hallström & Schönborn, 2019). Mathematical modeling concerns mathematization of real-life problems and provides mathematical solution to interpret the phenomena studied. Scholars have considered model and modeling to enhance authenticity (Anhalt & Cortez, 2016; Anhalt et al., 2018; Corum & Garofalo, 2019; Tran & Dougherty, 2014; Vos, 2018) in the STEM field (France, 2018). In the mathematical modeling classroom, students need to simplify a given real-world situation, explain, analyze, justify, identify, compare, reject, validate or revise mathematical solutions (Blomhøj & Jensen, 2003; Blum et al., 2007; English, 2003; Kaiser & Schwarz, 2006; Maaß, 2006). However, prior research documented that using mathematical modeling activities was difficult for pre-service teachers (Anhalt et al., 2018; Corum & Garofalo, 2019). For example, students perceived frustration in the first step of the modeling process, which was to make simplifying assumptions concerning the real-life task.

To enhance student success in modeling competency, the role of metacognitive competencies emerged as a promising way for positively affecting mathematics education (Desoete & De Craene, 2019) especially in the process of...
mathematical modeling competency (Galbraith, 2017; Kaiser & Stender, 2013; Vorbölder, 2018, 2019; Wendt et al., 2020). Metacognitive competencies are richly linked to modeling processes by means of a selected modeling task (Wendt et al., 2020). However, much of the research indicated that metacognitive behavior was a distinctive profile of competencies revealed by experts and novices in solving complex problems. Zimmerman and Campillo (2003) summarized that experts in problem solving displayed higher use of hierarchical knowledge when formulating strategic solutions, higher use of self-monitoring of strategies, more accurate self-evaluation, and higher motivation than novices. Conversely, novices were less active in employing metacognitive skills when solving problems (Blummer & Kenton, 2014). For example, in the planning stage, experts usually spend more time defining the problem or activating their prior knowledge (Brand-Gruwel et al., 2005). In addition, the most frequent part for experts than for novices is in regulating the problem-solving process. Novices did less monitoring and steering activities during the task performance and oriented themselves less often on the time left to carry out the task.

In a comparison of students according to the academic year level, several studies have indicated differences in metacognition among college students. Coskun (2018) found that students in the education field had lower metacognitive thinking skills, but they were much more capable in metacognitive strategies as the class level increased. Besides that, there seemed to be different roles of sub-dimension of metacognition at the beginning and end of the academic year. Hong et al. (2015) found that while there was no significant difference for knowledge of metacognition, there was indeed significant difference for metacognition regulation at the beginning and end of the academic year. In terms of mathematical modeling competency, various research documented that mathematical modeling competency was also difficult for university students (Hidayat & Iksan, 2018; Huang, 2018). Based on work by Fu and Xie (2013), for example, through one semester of research, freshmen made no progress in mathematical modeling competency. Therefore, researchers should pay more attention to student ability to make logical decisions about real-world problems, to understand objectives of modeling problems, and to define the parameters, variables and constants required.

It is very important to understand factors that can help to boost mathematical modeling competency, but it is still uncertain if the significant factors that can foster growing competency in mathematical modeling are important. Schukajlow et al. (2018) recommend research on moderating factors which might influence mathematical modeling competency. Moreover, it is a very interesting field to investigate whether differences in modeling skills exist among students at different levels of education and what they are (Fu & Xie, 2013). Nevertheless, the moderating effect of academic year level on the direct relationship between metacognition and mathematical modeling competency has been recorded in only a few studies. To the authors’ knowledge, the moderating impact of academic year level on the direct correlation between metacognition and competency in mathematical modeling has not been evaluated yet. The present investigation expands the literature on mathematical modeling by exploring the interrelationships between metacognition and mathematical modeling and the level of the academic year as a moderator impact. For this purpose, two different research questions were examined via structural equation modeling (SEM). Specifically, this research hypothesized that:

1. Metacognition has a positive influence on students’ mathematical modeling competency.
2. The level of the academic year significantly moderates the metacognition and mathematical modeling relationship.

**Literature Review**

According to Germain-Williams (2014), there is no standard framework for mathematical modeling. However, the literature of mathematical modeling competency indicated metacognition (Kaiser & Stender, 2013; Schaap et al., 2011) is the important aspect for mathematical modeling. The relationship between mathematical modeling competency and metacognition would be explained by Realistic Mathematics Education (RME) Theory. In RME theory, students have to find their own knowledge under the process of guided reinvencion which required students to mathematize their own mathematical activity (Gravemeijer & Doorman, 1999). Freudenthal called this process mathematizing or mathematization (Gravemeijer & Terwel, 2000) formulated into two ways of mathematizing in an educational context (Treffers, 1978; Treffers & Goffree, 1985). Both the process of horizontal and vertical mathematization usually come about through action and reflection (De Lange, 1987). The concept of reflection in RME theory proposed by De Lange is aligned with the mathematical modeling steps by Stillman et al. (2007). For Stillman (2011), metacognition is closely related to the transition between stages in the modeling process. It is not only beneficial but also important for developing modeling competencies (Blum, 2011; Kaiser & Stender, 2013).

To Freudenthal (1991), horizontal mathematization leads from the world of life or reality to the world of symbols or mathematics. Likewise, students’ activity in horizontal mathematising is to identify specific mathematics in a general context, schematize, formulate and visualize a problem in different ways, discover relations and regularities, and transfer the real world problems to a mathematical problem and model (De Lange, 1996). In contrast to horizontal mathematizing, vertical mathematizing refers to the mathematical processing and refurbishing of the real world problem transformed into mathematics (Treffers & Goffree, 1985). To Freudenthal (1991), vertical mathematization is the process of interpretation mechanically, comprehendingly, reflectingly from mathematical world back into the world.
of life or reality. In brief, it focuses on moving within the abstract world of symbols (Van den Heuvel-Panhuizen & Drijvers, 2014). However, during all levels of mathematical development, vertical mathematization indicated close interaction toward horizontal mathematization and both are regarded as being of similar value. Horizontal and vertical mathematizing are reflexively related, and not seen as dichotomies (Rasmussen et al., 2005) or intertwined (De Lange, 1987).

**The Process of Modeling Competency**

Modeling processes are classified into six perspectives namely; realistic or applied modeling, contextual modeling, educational modeling with a didactical or conceptual focus, socio-critical modeling, epistemological or theoretical modeling and meta-perspective (Haines & Crouch, 2010). The current study falls under the educational perspective on mathematical modeling. Furthermore, Blomhøj (2009) states that discussion about a model, modeling, the modeling cycle, modeling competence, and applications is a prominent aspect in research under this perspective. Since modeling process has been variously used in the literature (De Lange, 2006; Galbraith & Stillman, 2006; Kaiser & Schwarz, 2006; Lesh & Doerr, 2003; Verschaffel, 2002), the current study falls under the educational perspective on mathematical modeling.

One of the good examples of research within the educational perspective on mathematical modeling is the work by Stillman et al. (2007). In this research, the students in fact start the modeling process with a messy, real-world situation. Then the context is understood, structured, simplified, and interpreted to achieve the real world problem statement. From this stage, they need to assume, formulate, mathematize to build a mathematical model. This process continues to come up with working mathematically, then interpreting mathematical output for obtaining the real world meaning of solution. When achieving this stage, it is important for the student to compare, criticize, and validate their result in order to revise the model or accept the mathematical solution. Finally, the processes culminate either in the report of a successful modeling result, or a further cycle of modelling if the evaluation shows that the solution is unsatisfactory in some way.

In line with the concept of modeling which refers to the modeling process, mathematical modeling competence also involves the willingness to complete tasks with mathematical aspects taken from reality through mathematical modeling (Kaiser & Schwarz, 2006). Other researchers have also documented that mathematical modeling competencies include the skills and competencies to carry out appropriate and goal-oriented modeling processes, or known as affective goals (Sekerak, 2010), as well as the willingness to put them into action (Maaß, 2006). In brief, the definition of mathematical modeling competence involves cognitive competence, affective competence and metacognitive competence (Biccard & Wessels, 2011). This definition seems ambiguous because it includes affective and metacognitive parts (Frejd & Årlebäck, 2011). It can be concluded that mathematical modeling competencies not only involve some competencies relevant to the mathematical modeling process but also involve the purpose of achievement and a positive attitude to perform the modeling process. However, this study only refers to cognitive competence; it investigates the process of mathematical modeling competency.

**Metacognition**

According to Maaß (2006), a prominent factor in developing modeling competencies is metacognition, which is not linear or unidirectional. Metacognition is higher-order thinking (Lesh & Zawojewski, 2007) involving the concept of psychological and cognitive (Papaleontiou-Louca, 2008). According to Flavell's (1979) model, metacognition is indicated by four major aspects namely metacognitive knowledge, metacognitive experiences, goals (or tasks), and actions (or strategies). Metacognitive knowledge contains knowledge or belief factors or variables, namely person, task, and strategy, which act and interact in ways to influence the course and outcome of cognitive enterprises. However, metacognitive knowledge about learning processes can be right or wrong, and this self-knowledge is usually quite resistant to transformation (Veenman et al., 2006). In terms of the person factor, it involves everything that one believes about the nature of self and other people as cognitive processors. In terms of task factor, it includes one's existing information during a cognitive enterprise while the strategy factor consists of the procedural knowledge escalating opportunity of attaining task goals.

Livingston (2003) defined metacognition as an active control over the cognitive processes engaged in the learning process. Schraw and Moshman (1995), distinguished metacognition into two basic categories: metacognitive knowledge and metacognitive control processes. Metacognitive knowledge also known as knowledge of cognition is defined as what students know about their own cognition or about cognition in general. Knowledge in this context would also include beliefs, whether factual or not (Garofalo & Lester, 1985; Schoenfeld, 1983; Stillman & Galbraith, 1998). The three main types of metacognitive knowledge involves declarative knowledge, procedural knowledge, and conditional knowledge (Schraw & Moshman, 1995). Declarative knowledge refers to knowledge about oneself as a student and about what variables affect one's achievement. Procedural knowledge includes knowledge about the carrying out of procedural skills. Conditional knowledge involves knowing when and why to apply a variety of cognitive actions. At the same time, metacognitive control processes also known as regulation of cognition or metacognitive skills are defined as metacognitive activities which assist in controlling students’ thinking or learning (Schraw & Moshman,
1995) which incorporates planning, monitoring, and evaluation. Planning includes selecting compatible skills, strategies, and allocating resources in order to solve a problem. Monitoring activities involve students’ consciousness of comprehension and task achievement. Lastly, evaluation activity is defined as assessing the products and regulatory processes of one’s learning which include goals and conclusions.

Although Wilson and Clarke (2002) generally recognize the benefits of metacognition in the learning process, little is known about the types of metacognitions that help. According to Maaß (2006), an important factor in developing effective modeling competency is metacognition, which is not linear or one-way. Metacognition involves the process of management and coordination; it is very important to solve problems that involve complex activities such as various cognitive operations (Garofalo & Lester, 1985). Metacognition guides students to select strategies in understanding tasks or problems, plan actions, monitor implementation activities, evaluate the results of strategies and plans and while revising or abandoning unproductive strategies and plans (Brown, 1978). For example, in the modeling cycle it can be used to identify the type of intervention needed to overcome certain obstacles (Stillman, 2011). According to Lingefjärd (2011), metacognitive competencies that encompass the process of mathematical modeling are important to be involved in the framework of obstacles and opportunities.

**Empirical Evidence on Metacognition and Modeling Competency**

Since metacognition is the most vital factor of mathematical achievement (Desoete et al., 2019; Tian et al., 2018; Tjalla & Putriyani, 2018), scholars in mathematical modeling (Galbraith, 2017; Kaiser & Stender, 2013; Maaß, 2006; Stillman, 2011) suggested that it is also a prominent factor in developing mathematical modelling competency. Yildirim (2010) documented that all sub-constructs of metacognitive factor significantly contribute to the development of student modeling strategies. Moreover, based on research by Vorhölder (2019) who evaluated the students and group level, the findings indicate that in the experimental group, the teaching unit resulted in a substantial increase in evaluation strategies, but not in the control group and not according to strategies for proceeding and regulating. Students from the metacognition group reported that at the end of the study, they employed strategies for evaluation substantially more often than before. This is in line with Stillman (2011) who states that only certain metacognitive actions are productive. According to him, there are three stages in which productive metacognitive action develops, namely recognition, where certain strategies are relevant, strategy choices for implementation, and successful implementation.

Hidayat et al. (2020) evidenced that meta-cognitive behavior has significant and positive relationship with mathematical modeling. Metacognitive behavior is important for students when they improve models during the process of mathematical modeling competency such as justification (Sharma, 2013). Kramarski et al. (2002) found that the presence of metacognitive questions is quite interesting in modeling activities. For example, students can explain further, at the same time, metacognitive questions may guide students to find all the important information, distinguish between relevant and irrelevant information and understand the whole problem rather than part of it. Recent work of Rellensmann et al. (2020) documented that enhancing strategic knowledge, which refers to metacognitive knowledge, about drawing, particularly among non-high-achieving students, could be a way to drawing and modeling success. In addition, this kind of metacognitive knowledge was discovered to be linked to drawing efficiency modeling achievement even when controlling for cognitive skills and interest. Finally, previous investigation concluded that although only certain sub-constructs of metacognitive strategies are productive, metacognitive strategies are a powerful predictor for the process of mathematical modeling competency with different effect. Therefore, the model integrating these constructs has not been tested previously, based on previous work, and the fit of the current structure is evaluated using structural equation modeling (SEM). The proposed moderated model which integrates metacognition and mathematical modeling that might be affected by academic year level is developed by theories and prior investigations (Figure 1). We assumed that the academic year level has moderation effects on the direct relationship between metacognition and mathematical modeling competency. The academic year level would affect the strength of the relationship between metacognition and mathematical modeling competency.

![Figure 1. The Proposed Moderated Model](image-url)
Methodology

Participants and Procedures

We used a correlational research design to answer research questions of the study. Cluster random sampling was adopted to gather the sample from the population of mathematics education programs in Riau Province, Indonesia, which had the same modeling experiences. The participants were randomly chosen from three out of six universities. We sent consent letters to 538 students to participate in this study. The consent letter introduced the purpose of the current study after being approved by the Department of Investment and Integrated One Stop Services, Indonesia. Among the 538 valid participants, 89.8% were female and 10.2% were male. Out of 538 participants, 133 students (24.7%) were from the first academic year, 223 (41.4%) from the second, and 182 (33.8%) were from the third. They were given 60 minutes to fill out a mathematical modeling test and metacognitive instrument during the lecture hours voluntarily.

Research Instruments

The mathematical modeling test was adopted from Haines and Crouch (2001) and includes eight sub-constructs to gauge mathematical modeling competency. The mathematical modeling test consists of 22 multiple-choice questions; the true answers were awarded 2 scores, partially true answers were awarded 1 score, and incorrect answers were awarded 0 score. Many researchers have used mathematical modeling competency instruments with various objectives to measure students’ mathematical modeling competencies in both secondary and tertiary education students (see Frejd & Årlebäck, 2011; Fu & Xie, 2013; Hidayat et al., 2021; Kaiser, 2007). In our sample, the reliability coefficients for these sub-constructs were .87, .82, .81, .72, .76, .86, .73, and .75 respectively. The reliability coefficients of the mathematical modeling test as a whole were good (.82) (Tavakol & Dennick, 2011). Moreover, the mathematical modeling test of composite reliability (CR) for all indicators ranged from .73 to .88, thus implying that all the indicators were higher than the .6 desired standard, demonstrating a high internal consistency. The mathematical modeling test of the Average Variance Extracted (AVE) ranged from .50 to .70 which were higher than the .5 benchmark, showing a good discriminant validity.

The meta-cognitive inventory was adopted from O’Neil and Abedi (1996), which was modified by Yildirim (2010) for mathematical modeling context and includes four sub-constructs to evaluate metacognition. The metacognition instrument includes 20 items with five-point scale (1 = strongly disagree, 2 = disagree, 3 = uncertain, 4 = agree, and 5 = strongly agree). In our sample, the reliability coefficients for these sub-constructs were .83, .85, .84 and .83 respectively. The reliability coefficients of the meta-cognitive inventory as a whole were above the α > .70 criterion, which is good (Tavakol & Dennick, 2011). In addition, the metacognitive inventory of composite reliability (CR) for all indicators ranged from .83 to .85, thus indicating that all the indicators were higher than the .6 minimum common cut-off, demonstrating a high internal consistency. At the same time, the meta-cognitive inventory Average Variance Extracted (AVE) ranged from .50 to .54 which were higher than the .5 minimum common cut-off, showing a good discriminant validity.

Data Analysis

Before doing further research, this study examined a wide variety of data screening-related concerns, such as addressing missing value, multi-collinearity, and identifying outliers and normality (Mohamed & Rosli, 2014). We employed structural equation modeling (SEM) to test the hypothesized moderation employing IBM SPSS Amos version 18. The flexibility of SEM allows it to be used in a variety of study designs, involving experimental and non-experimental data, cross-sectional and longitudinal data, and data from different groups and levels (Kwok et al., 2018) such as research conducted by Qin et al. (2019). A measurement model (Confirmatory Factor Analysis-CFA) was computed to examine whether the dimension structures of the instruments would be confirmed for the sample in present study. The first CFA was calculated for the metacognition consisting of four dimensions (planning, cognitive strategy, self-checking and awareness). In the second CFA, the eight-factor mathematical modeling competency (simplify assumptions, clarify aims, formulate problem, assign variables, formulate mathematics, select a model, interpret graph and relate mathematical solution) was examined. The assessment of model adequacy was determined based on the score of chi-squares ($\chi^2$) ($p > 0.05$), normed chi-square ($\chi^2 / df$), comparative fit index (CFI > 0.90), root-mean-square error of approximation (RMSEA < 0.08), Tucker-Lewis index (TLI > 0.90) and adjusted goodness-of-fit index (AGFI > 0.90) (Mohamad et al., 2018). At the same time, we also computed composite reliability (CR), Cronbach’s alpha coefficients and average variance extracted (AVE) to determine convergent and discriminant validity and the reliability of the measures. For Hair et al. (2010), alpha scores of .60 to .70 are acceptable while CR must be over .60 and AVE must be higher than 0.50 (Mohamad et al., 2018). To further examine the moderating impact of academic year level in the a priori models, the participants were categorized into year one ($n = 133$), year two ($n = 223$) and year three ($n = 182$) subgroups. Multigroup analysis was conducted to explore the moderating effect of academic year level on the direct relationship between metacognition and mathematical modeling competency.
Results

Descriptive Statistics

The descriptive statistics explained means, standard deviations, skewness, kurtosis and the correlations among constructs. The descriptive statistics and the correlations among variables are indicated in Table 1.

Table 1. Descriptive Statistics and the Correlations Among Dimensions

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<td>Clarify aim</td>
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<td>Formulate Problem</td>
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<td>Assign Variable</td>
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<td>.146**</td>
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<td>Formulate mathematics</td>
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<td>.086*</td>
<td>.300**</td>
<td>.200**</td>
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<td>Select Model</td>
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<td>.230**</td>
<td>.096*</td>
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<td>Interpret graph</td>
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<td>.148**</td>
<td>.213**</td>
<td>.107*</td>
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<td>.206**</td>
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<td>.085*</td>
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<td>.237**</td>
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<td>-.852</td>
<td>.842</td>
<td>1.343</td>
<td>.087</td>
<td>.106</td>
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SD: Standard deviation; *p<0.05; **p<0.001

Overall, descriptive outputs indicated a significant and a low and moderate level of correlation among sub-constructs of mathematical modeling competency and sub-construct of metacognition. Clarifying the aim of the real model was found to have the lowest correlation with formulate relevant mathematical statements ($r = 0.086$, $p < 0.05$). Conversely, planning was moderately and positively linked to cognitive strategy ($r = 0.500$, $p < 0.001$). Moreover, the mean scores varied among sub-constructs of mathematical modeling competency; simplifying assumptions about the real-world task ($M = 1.116$, $SD = .692$) was the highest and clarifying aim of the real model ($M = .734$, $SD = .626$) was the lowest mean value. In terms of normality, skewness scores in each sub-construct were relatively low which ranged from -.658 to -.496, whereas kurtosis scores range from -1.195 to 1.343 for all sub-constructs, revealing that all constructs were normally distributed in the current study. In terms of multivariate normality [critical ratio of multivariate kurtosis: mathematical modeling competency = 2.434; and metacognition = 33.349]. Since the data set for metacognition were not normally distributed, the bootstrapping procedure was employed (Hayes, 2009). Moreover, outliers and missing are not found for each construct in the present research.

Measurement Model

Results of the CFA indicated that the mathematical modeling competency with eight sub-dimensions had very good model fit for Indonesian settings; $\chi^2 = 262.179$, $\chi^2/df = 1.304$, CFI = 0.975, GFI = 0.958, AGFI = 0.947, TLI = 0.971 and RMSEA = 0.024. All factor loadings of the mathematical modeling competency sub-constructs ranged from 0.602 to 0.832, which surpassed the cut-off values of 0.50 (Hair et al., 2010). At the same time, CFA outputs indicated that metacognition with four sub-dimensions had provided an acceptable measurement model fit for Indonesian settings; $\chi^2 = 335.891$, $\chi^2/df = 2.023$, CFI = 0.963, GFI = 0.943, AGFI = 0.927, TLI = 0.957 and RMSEA = 0.044. All factor loadings of the mathematical modeling competency sub-constructs ranged from 0.636 to 0.780, exceeding the cut-off values of 0.50 (Hair et al., 2010).

Structural Model

The outputs of maximum likelihood estimation revealed that the structural model fit the data well for Indonesian settings; $\chi^2 = 1151.259$, $\chi^2/df = 1.428$, CFI = 0.973, GFI = 0.909, AGFI = 0.900, TLI = 0.949 and RMSEA = 0.028. Goodness-of-fit indices were consistent with the cutoff model-fit value suggested by Mohamad et al. (2018). Factor loadings in the structural model ranged from 0.630 to 0.773 for metacognition; and ranged from 0.615 to 0.835 for...
mathematical modeling competency. All factor loadings in the structural model exceed the cutoff model-fit criteria of 0.50 (Hair et al., 2010) and were statistically significant, \( p < .05 \). The path analysis outputs revealed that the recommended regression model was suitable, and metacognition was a significant predictor of the mathematical modeling competency (\( \beta = 0.462, p < 0.001 \)). Metacognition accounted for variance of 78% (\( R^2 = 0.78 \)) in mathematical competency. Students who employ meta-cognitive behavior in modeling activities accomplished well in mathematical modeling competency. Students’ metacognition was important in enhancing learners’ mathematical modeling competency. Figure 2 indicates the hypothesized model, goodness-of-fit indices and standardized factor loadings. Moderation impact of academic year level on the relation between metacognition and mathematical modeling competency were computed for following analyses.

**Figure 2. The Hypothesized Model**

**Moderation Impact of Academic Year Level on The Relation between Metacognition and Modeling Competency**

We hypothesized that the academic year level had moderation effects on the direct relationship between metacognition and mathematical modeling competency. The moderator effect existed if at least one group had a Chi-square value difference above 3.84 (Awang et al., 2018). Table 2 revealed the moderation test for year one, two and three which refer to the constrained and unconstrained model.
As seen in Table 2, academic year level moderates the relationship between metacognition and mathematical modeling competency. The difference in Chi-Square score for year two and three between the unconstrained and constrained model was more than 3.85 (Awang et al., 2018), indicating a significant moderating impact. Moreover, Table 3 indicated the effect of the moderator in which group (year one, year two or year three) was more pronounced on the direct relationship between metacognition and mathematical modeling competency.

Table 3 indicated standard regression coefficient value for year one, year two and year three group on the direct relationship between metacognition and mathematical modeling competency. The estimated standard regression coefficient value was 0.351. Moreover, the result revealed that the type of moderator effect was partial moderator since all regression coefficients for each group were significant (Awang et al., 2018). Therefore, the research findings indicated that academic year level influenced the strength and direction of the interrelationship between metacognition on the mathematical modeling competency. The effect of metacognition on mathematical modeling competency was more pronounced in the year two group compared to the year one and three group.

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Table 3. Standard Regression Coefficient Value

<table>
<thead>
<tr>
<th>Year</th>
<th>Modeling competency</th>
<th>Metacognition</th>
<th>Estimate</th>
<th>SE</th>
<th>CR</th>
<th>p</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year one</td>
<td>Modeling competency</td>
<td>Metacognition</td>
<td>.402</td>
<td>.160</td>
<td>.062</td>
<td>2.567</td>
<td>.0100</td>
</tr>
<tr>
<td>Year two</td>
<td>Modeling competency</td>
<td>Metacognition</td>
<td>.881</td>
<td>.643</td>
<td>.129</td>
<td>4.970</td>
<td>****</td>
</tr>
<tr>
<td>Year three</td>
<td>Modeling competency</td>
<td>Metacognition</td>
<td>.351</td>
<td>.153</td>
<td>.078</td>
<td>1.968</td>
<td>0.049</td>
</tr>
</tbody>
</table>

Driven by RME theory and empirical evidence on metacognition and modeling competency, the aim of this research was to explore the direct interrelationships between metacognition and mathematical modeling competency and the level of academic year as a moderator via SEM approach. It is important to study whether and how the level of the academic year influence the interrelationship between metacognition and mathematical modeling.

More notably, our research confirms that the direct correlation between metacognition and mathematical modeling was observed to be statistically significant. This conclusion is in line with scholars’ suggestion (Galbraith, 2017; Kaiser & Stender, 2013; Maas, 2006; Stillman, 2011) and findings of prior investigation (Hidayat et al., 2018; Rellensmann et al., 2020; Vorhölter, 2019; Yildirim, 2010) indicating that metacognitive strategies are powerful predictors for mathematical modeling process competency. Metacognitive techniques are employed to help direct the learning process (Fathurohman & Cahyaningsih, 2021). This can be explained by Brown (1978) who said that metacognitive strategies can guide someone to choose suitable strategies in understanding a complex or messy problem in mathematical modeling. Moreover, implementing metacognition also helps students to monitor implementation activities, assess the outputs of strategies and plans while revising or abandoning unproductive strategies and plans. The cognitive process of seeing a model has a significant impact on problem-solving efficacy (Anoling et al., 2018). Since the concept of metacognition in the proposed framework is closely related to the transition between stages in the modeling process, Stillman (2011) believed that metacognitive strategies can be utilized to recognize the type of intervention required to cope with certain barriers. Another possible reason for the current result in our investigation is that students can be directed by metacognitive queries to discover all the crucial information, differentiate between accurate and inaccurate data, and comprehend the whole issue rather than part of it. This is line with statement with Sawuwu et al. (2018) indicating that one of the sub-constructs of metacognitive knowledge on how to handle an issue successfully was procedural knowledge.
The moderating effect of academic year examined in the research question was clearly revealed. Academic year level significantly moderates the interrelationships between the metacognitive strategies and mathematical modeling competency. In the second academic year category, the impact of metacognition on mathematical modeling competency was more pronounced compared to the first academic year and the third academic year group. The significance of the roles of metacognitive strategies on mathematical modeling competency has consistently been evidenced in prior works (e.g., Coskun, 2018; Hong et al., 2015). As mentioned in the introduction, the possible explanation why the second academic year category might be more effective in moderating the effect of metacognitive strategies on mathematical modeling competency than the first academic year group relates to level of metacognitive thinking skills. As the class level grows, students are also much more aware of metacognitive strategies. The second academic year students are more aware of metacognitive strategies than the first academic year group, indicating that they hold better level of mathematical modeling. Another possible explanation is that students in different level have faced learning and progress surroundings, thus having different chances to be aware of their cognitive development.

However, it is difficult to explain why the role of metacognitive strategies on mathematical modeling competency was noticeably lower for the third academic year group compared to others. The finding of current research can be explained by previous work that mentioned that not all sub-constructs of metacognitive strategies were not significant. This is consistent with view of Wilson and Clarke (2002) who indicated that very little is known of the aspects of metacognition that assist. For instance, Hong et al. (2015) found no significant difference for knowledge of metacognition at the beginning and end of the academic year. At the same time, Vorhölter (2019) also argued that students used strategies for evaluation significantly but not strategies for proceeding nor for regulating. Another reason might be related to difficulty of mathematical modeling. Students’ failure to translate inquiries in word problems, such as failing to generate crucial information or failing to pick the proper symbols or writing methodically, is one of their shortcomings (Che Md Ghazali et al., 2019). Although the effect of metacognitive strategies was less pronounced for the third academic year category, the moderating effect of academic year level was still significant between metacognitive strategies and mathematical modeling. Overall, it can be concluded that there is significant moderating impact of academic year level between metacognitive strategies and mathematical modeling in which the second academic year group had a higher estimated standard regression coefficient. Students in the second academic year group felt a more beneficial effect of metacognition toward mathematical modeling competency than other categories.

Conclusion

Several concerted movements toward mathematical modeling have been witnessed in the last decade, reflecting the growing global relationship between the role of mathematics in the context of modern science, technology and real life. More consideration has been devoted to the theoretical basis of research questions in the literature on mathematical modeling and to the use of effective research methods in the studies. In our findings, the metacognitive strategies are a powerful predictor (78%) for the process of mathematical modeling competency. At the same time, the moderating role of the academic year level explored in the study question was clearly observed. In contrast to other categories, the role of metacognition on mathematical modeling competency was more pronounced. Consequently, through metacognition, lecturers can influence the level of mathematical modeling competency among university students. Considering the role of academic year level as moderator, lecturers can allow students to maximize their metacognition in term of boosting mathematical modeling for the second year. At the same time, for other groups, lecturers can enable students to be aware of the power of metacognition during mathematical modeling activities for the prospective secondary mathematics teachers. Finally, the proposed moderated model might be one of the directions for future study to increase students’ mathematical modeling competency.

Recommendations

Upcoming research should pay close attention to research design (e.g., experimental design), to capture the effect of metacognition on mathematical modeling. It is also necessary to develop new tests to measure mathematical modeling competency for the prospective secondary mathematics teachers. Since mathematical modeling courses for students of mathematics education program are not formally introduced in the curriculum (Widjaja, 2013), lecturers need to be aware of examples of mathematical modeling in another course. For Widjaja (2013), it is important to engage preservice teachers first-hand as mathematical modelers before expecting them to promote mathematical modeling in their own teaching. Lastly, since the present findings discovered that there was significant moderating effect of academic year level between metacognition and mathematical modeling, it is suggested that future research widen the sample to include high school students, primary and secondary mathematics teachers.

Limitations

Since the current study employed correlational design via SEM, it is quite difficult to explain the causal effect of metacognition on mathematical modeling competency. Furthermore, the instrument of mathematical modeling employed in the current research was not developed from the educational perspective especially for the prospective secondary mathematics teachers. In terms of sampling, the study only employed limited sample in one province in
Indonesia; conducting a big project of mathematical modeling using random sampling of the prospective secondary mathematics teachers would enhance our knowledge of teaching and learning of mathematical modeling competency. Again, the selection of the moderating and mediating effect for mathematical modeling competency should be carefully considered.

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Competing Interests
Authors declare no personal relationship(s) that might have influenced them inappropriately during the writing of the present study.

Authorship Contribution Statement

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