Using Reciprocal Teaching for Improving Students’ Skills in Mathematical Word Problem Solving - A Project of Participatory Action Research

Abstract: This study aims to present the potential of Participatory Action Research (PAR) to bring together the experiences of teachers and researchers with the intention of improving teaching practices and students’ learning outcomes. Participants in the study were 7 teachers, their 160 fifth grade students, and researchers (authors). Teachers and researchers participated as partners in all collaborative activities during the period of 12 weeks. All teachers assisted by the researcher (first author) who serves as a teacher at the same school, were involved in implementing the reciprocal teaching method (RTM) in math classes. They examined each step of the implementation of this method in order to investigate whether it has an impact on student achievement in solving mathematical word problems. Teachers observed the work of students in their classes, whereas in the joint meetings they discussed occasional ambiguities as well as issues that were most challenging for them and their students. The results showed that there was a significant improvement of the students’ results in the post-test of the mathematical word problems. The analysis of teachers’ reflections highlights the benefits of collaboration within the PAR project, both for students and teachers. The study suggests that the PAR model can be used effectively within school settings as a research model, and as a pedagogical practice.

Keywords: Reciprocal teaching, mathematical word problems, participatory action research.

Introduction

Problem solving is one of the most important competencies that requires special attention at all levels of education. According to the Kosovo Curriculum for Pre-university Education, it enables the development of students’ skills and abilities for critical thinking (Ministry of Education, Science and Technology, Republic of Kosovo, 2016). Moreover, it enables independent and systematic work encouraging curiosity to build new knowledge. Learning to solve problems begins with learning abstract concepts and then shifting towards problem solving to apply the ‘explain-practice-apply’ skills (Van de Walle et al., 2016). This way, students’ attention is focused on ideas to create an understanding of the problem, develop mathematical processes, beliefs, and provide an accurate solution to problems. However, for most students, problem solving is a complex and difficult process, especially when they face word problems. Thus, it is important to start developing the word problem solving skills early in elementary school, as word problems get semantically more complex when students’ progress in their future educational levels (Boonen et al., 2016).

The findings of the studies (Salihu & Räsänen, 2018; Vula et al., 2017) have shown that a significant number of primary school students in Kosovo have difficulties in the subject of mathematics in general, and particularly in solving mathematical word problems. These studies suggest paying more attention to teaching practices, enabling all students to improve their skills in mathematics. Since many other countries face same problems related to learning mathematics, the researchers of the present study made attempts to investigate not only effective teaching practices, but also the manner of their implementation with students.

In this PAR project, a formal collaboration is initiated between teachers and researchers to explore whether teachers’ commitment to transforming their teaching practices has an impact on improving students’ achievement in solving math word problems. The reciprocal teaching method (RTM) adapted by Van Garderen (2004) was used as a method for solving math word problems, while participatory action research (PAR) was used as a research design that enables collaboration within the project.

* Corresponding author:
Eda Vula, University of Prishtina “Hasan Prishtina”, Faculty of Education, Prishtina, Kosovo. edavula@uni-pr.edu

© 2021 The Author(s). Open Access - This article is under the CC BY license (https://creativecommons.org/licenses/by/4.0/)
Literature Review

Mathematical word problems

Word problems refer to “any mathematical exercise where significant background information on the problem is presented as text rather than mathematical notation” (Boonen et al., 2013, p.271), that prepare students to practice mathematical skills in everyday situations (Pongsakdi et al., 2020). Word problems help to make the connection between everyday language, mathematical vocabulary, and mathematical operations. In their study, Pongsakdi et al. (2020) pointed out that practicing more demanding mathematical word problems is not only beneficial for learning mathematics, but can be an effective way for acquiring more advanced reading comprehension skills. However, many studies have shown that problem solving in general, and particularly mathematical word problem solving, is a difficult process for students and often for teachers themselves (Daroczy et al., 2015; Jitendra et al., 2007; Özsoy et al., 2015; Pongsakdi et al., 2020; Zhu, 2015). As a complex cognitive activity, word problem solving is often considered problematic for students, usually when they are presented as a story or scenario from real life. In these cases, their solution involves several processes and strategies. It requires the use of linguistic information, identification of necessary data, and determination of methods for finding solutions (Vula et al., 2017). Meanwhile, as Zhu (2015) argues, the mathematical word problems represent a series of actions required by students: they need to apply many activities, and a high-level thinking, not just to memorize facts or practice some routine procedures. Hence, the difficulties that students usually face are particularly related to: 1) transformation of verbal problems into mathematical problems; 2) misunderstanding of the purpose of the word problem; 2) lack of mastery of various problem-solving strategies; 3) lack of mastery of the concepts that are needed to solve the problem; 4) a practical lack in reviewing the results of calculations (Nurhayanti et al., 2020). There are many studies that have identified a number of the sources of these difficulties. Most of them are interrelated with the semantic structure of word problems and the most frequent reason that makes these problems so difficult is the comprehension of their content (Capraro et al., 2012; Vula et al., 2017). Even the study of Pearce et al. (2013) showed that teachers’ perspectives on the difficulties that students have on word problem solving are related to students’ ability to read and understand the problem.

Partial understanding of problem information and requirements can often be an obstacle to solving them. The other studies have shown that difficulties of word problem solving increase when inconsistent language is involved, which creates a greater linguistic complexity (Boonen et al., 2016). What creates gaps in this kind of word problems is the failure to take into consideration the reciprocal relations in cognitive processes between reading and using mathematics (Vula et al., 2017). Furthermore, students frequently approach the problem in a procedural way. They mostly translate words directly into a math expression and make calculations without a mental representation of the problem (Pape & Smith, 2002). On the other hand, there are many cases when students who have difficulties with mathematical word problem solving are able to solve equivalent numerical problems, including accurate mathematical operations and accurate mathematical calculations (Reilly et al., 2009). In these cases, even though students may know the procedures for solving numerical problems, they may have difficulties with integrating different cognitive processes, or maybe they have limited memory capacity (Jitendra et al., 2007). Such difficulties according to Montague et al. (2011) often appear also due to the involvement of multiple cognitive processes in the word problem solving process. Therefore, the emphasis only on understanding the “text” involved in the word problem is not sufficient. In addition, both factors, the linguistic factor, as well the numerical one, are important factors that should be considered when solving word problems (Daroczy et al., 2015). Developing students’ skills for a deeper text comprehension of the word problems should serve as a crucial step before the correct mathematical computations can be performed (Boonen et al., 2016).

In recent years there are studies that address the issues of word problems by examining different strategies and methods which can help students to overcome the difficulties in involving them (Fuchs et al., 2004; Jitendra et al., 2007; Özsoy et al., 2015; Zhu, 2015). Dealing with information about semantic relations between the given unknown quantities is frequently a difficult process for most of the students. Hence, researchers have focused on examining the classroom interventions using reading comprehension skills (Boonen et al., 2016; Capraro et al., 2012; Glenberg et al., 2012; Kurshumlia & Vula, 2019; Limond, 2012; Van Garderen, 2004; Wessman Huber, 2010). The other researchers have used intervention programs that focus on both strategic mental representation skills and reading comprehension skills (Fuchs et al., 2004; Jitendra et al., 2007; Montague et al., 2011; Vula et al., 2017). In addition, another group of researchers ‘have called repeatedly for the adoption of alternative pedagogical approaches that promote problem solving, discussion, and collaborative learning’ (Wright, 2021).

Reciprocal teaching as a teaching and learning method for math word problem solving

Reciprocal teaching is an effective methodology for developing reading comprehension (Palincsar & Brown, 1984). It is a cooperative learning methodology, where a small group of students apply four reading strategies (questioning, clarifying, summarizing, and predicting) to co-construct the meaning of a written text (Tarchi & Pinto, 2016). Hartman (1994) emphasizes that scaffolding instruction and collaborative learning, as the main activities of reciprocal teaching, enable the development of cognitive, and metacognitive strategies during the reading process. As an interactive method, reciprocal teaching involves group dialogue, led by the group leader, and consists of four strategies: prediction,
clarification, questioning, and summarizing. The reciprocal teaching is able to create an interaction-rich and diverse environment (Tarchi & Pinto, 2016). The reciprocal teaching is designed to improve fundamental reading comprehension strategies (Hartman, 1994), but it can also be used in other courses. Since it is flexible and adaptive for different levels of students, reciprocal learning is quite effective in supporting students in solving mathematical word problems (Meyer, 2014; Pilten, 2016; Reilly et al., 2009; Van Garderen, 2004; Wessman Huber, 2010).

During reciprocal teaching the focus is on a) teaching students about concrete strategies on fostering understanding while they read the text, and b) this learning can take place mainly in the context of a dialogue between the teachers and students (Rosenshine & Meister, 1994). Accordingly, this method is characterized by a dialogue between teacher and student or even among students themselves for the implementation of four strategies (Hartman, 1994; Palincsar & Brown, 1986; Woolfolk, 2011). The reciprocal teaching method in some cases can provide a way to develop a specific mathematical language (Meyer, 2014), and help students for developing self-regulating skills (Lenchuk, 2021). Moreover, the use of this method is able to create an interaction-rich and diverse environment (Tarchi & Pinto, 2016).

In the study that used the reciprocal teaching method, Van Garderen (2004) modified the RT method for solving math word problems. The study proposed the following strategies: clarification (clarification of words or expressions), questioning (identification of key terms of the problem), summarizing (purpose of the problem, the reason the mathematical problem is being solved), and planning (developing a plan and actions needed to solve the problem). Interactions are the key feature when reciprocal teaching is used for word problem solving (Van Garderen, 2004; Wessman Huber, 2010). Through the clarification and questioning strategies, students work together to list the words that they did not know, the facts that are known for them, and the information they had to determine (Meyer, 2014). The other strategies within the reciprocal teaching include: students’ work using pictures, diagrams, numbers, or words to summarize the aim of the word problem and, finally, planning the problem-solving strategies. At the last stage, students engage in self-reflection, including reasoning their response, reflecting on how they could refine their approach if faced with a similar problem, and assessing how they contributed to the problem-solving group (Van Garderen, 2004).

However, in order to develop students’ problem-solving skills, particular attention must be paid to teaching practices. Teachers need to continually review their professional development and find appropriate teaching methods that have an impact on improving students’ learning outcomes.

In their article, Robutti et al. (2016) presented the results from a survey that was focused on “Teachers Working and Learning Through Collaboration”. They explain that sometimes teachers are involved in collaborative settings that involve the research initiative where they engage in the joint activity for common purposes, and mutual support in addressing issues that challenge them. Participation research in itself is a powerful pedagogical perspective that provides participants an agentic means to effect contexts that can transform their teaching practices (Moreno & Rutledge, 2019).

Methodology

Research method

Participatory research is a powerful pedagogical perspective that provides participants with the ability to control his or her actions in order to transform their teaching practices. Hence, the present study used Participatory Action Research (PAR) as a research design (Richter, 2016). Leavy (2017) noted that it refers to a research method in which all participants serve in partnership, from the identification of the research problem to the final presentation of the results, and discussion of their action implications. By collaborative partnership “research is understood to be participatory and action-oriented” (Leavy, 2017, p. 234). Usually, in the educational environment, researchers, and teachers, cyclically design teaching interventions regarding a topic, where they have identified a gap in the curriculum, pedagogy practices, or students’ learning process. Thereby, they collaborate with each other and take systematic action to improve the issue of concern to them (Fraenkel et al., 2012). The important aspects of doing participatory research are the critical reflections and exploratory reasoning which were involved in the participants’ discussions (Skovsmose & Borba, 2004).

Research questions

The study addresses the following questions:

1. What is the impact of the reciprocal teaching method for improving students’ achievement in math word problem-solving?

2. How do teachers evaluate their involvement in the participatory action research project?
Sample and Data Collection

The research was conducted in a public primary school "Mustafa Bakija" in the municipality of Gjakova in Kosovo, with the students attending grades 1 to 5, and pre-school classes. All fifth-grade students (N = 160), 10-12 years old, were part of the study. The average age of students participating in the research was M = 10.16, and SD = .413.

For the purposes of this research project, all seven teachers of the fifth grades as well as the researchers (authors) were partners in all collaborating activities. The researchers were responsible for facilitating the teachers’ work, as well as leading the activities through all PAR cycles.

In order to measure the impact of the reciprocal teaching method in solving math word problems, the total of ten mathematical word problems were designed for the pre-test. They varied in complexity of the language, or in the number of arithmetic operations to be solved. Therefore, the problem ‘The library has 760 books for adults and 478 books for children. How many books are there in the library in total?’ is a simple word problem, with consistent language, and for solving it students can use one arithmetic operation. The problem, ‘A round trip plane ticket from Prishtina to Rome costs 260 euros. This price is 80 euros lower than the round-trip ticket from Prishtina to Paris. How much do four round-trip tickets from Prishtina to Paris cost?’ is more difficult because the language is inconsistent, and it can be solved using more than one arithmetic operation. The same procedure was repeated later once again after the intervention, with ten new problems, equivalent in measure with the problems of the pre-test.

All students completed their tasks using paper and pencil. Students’ work was assessed based on the rubric designed according to four criteria: i) understanding of the problem, ii) planning the solution of the problem, iii) problem solving, and iv) justifying the solution. Each criterion was evaluated with 0-2 points; therefore, the maximum number of points to be scored by a student for a solved problem was 8 points in total, while the maximum points for the pre-test and the post-test was 80 points in total. The reliability for the pre-and post-test (Alpha Cronbach) was 0.90 and 0.88, respectively.

On the other hand, reflective sheets for classroom observation were used by the teachers. They contained rubrics with notes on student work based on their roles (predictionary, explorational, questioner and summarizer), problem solving, as well as notes on achievements and challenges observed in class. These reflective sheets have been used for joint discussions between the teachers and the researchers. The data collected from the reflection sheets enabled researchers to examine the assessments of teachers involved in the PAR project.

Procedure

For all participants involved in the study and the majority of their students, the mathematical word problem solving was the most difficult process. Therefore, the researchers (authors), and participants involved in this PAR project, aimed to bridge this gap. The PAR also provided the opportunity for the participants to search for the literature, practice the new methods, and transform their teaching practices (Leavy, 2017). This PAR project was conducted in a collaborative, systematic, and rigorous way aiming to "carry out research with teachers (as research partners) rather than on teachers (as research objects)" (Wright, 2021). All activities within the PAR enable the researchers and teachers to act as full collaborators to empower themselves through constructing and using their own knowledge (Reason, 1994). In this research project, the researchers served as "guide facilitators, formulators, summarizers of knowledge, and as raisers of issues" (Cohen et al., 2007, p.301). On the other hand, teachers' responsibility was to use reciprocal teaching in their classrooms, increase the collaboration between them and students, and between their students during different phases of the word problem solving process. Also, they were responsible for observing the problem-solving process in their classes, and reflect on classroom actions in regular joint discussions, in which different issues were raised in order to find the best solutions in partnership with the researchers.

The PAR project enabled collaboration between researchers and teachers in identifying the issues that were of interest for them, in searching for the literature that informed them on intervention activities, and in reflecting upon them. The project also enabled them to discuss their experience with students, decide regarding the data collection, data analysis, and interpretation of the findings. According to McTaggart (1997) the triangulation of observation, reflections, and discussion was accompanied by the confirmation and coherence of the arguments presented in the community of "critical friends" (p.13), served as a tool for the validation of the whole research process. The PAR project realized in this study has been accompanied by actions, reflections, and measurements in all phases. Table 1 shows the cycle of realization of research phases based on the model modified by Kindon et al. (2007).
Table 1. The PAR cycles modified by Kindon et al. (2007)

<table>
<thead>
<tr>
<th>PAR cycles</th>
<th>Phases</th>
<th>Activities</th>
</tr>
</thead>
</table>
|            | Diagnosis    | - Presenting teachers’ concerns about their students’ gaps in word problem solving  
|            |              | - Conducting the pre-test to better understand students’ skills and their difficulties  
| First cycle| Measurement  | - Assessment of the pre-test and analyses of the students’ work  
|            |              | - Reflection on students’ work in the joint discussion relating to the difficulties that are identified  
|            | Reflection   | - Discussion on the reciprocal teaching method proposed by the researcher and decision for further actions  
| Second cycle| Action       | - Informing the teachers with the Reciprocal teaching method (RTM) and presenting instructions on how to implement this method by the researcher  
|            |              | - Implementing the RTM by teachers in their classes  
|            | Reflection   | - Reflective meetings for teacher observations during the implementation of RTM  
|            |              | - Reflections on experiences and other necessary clarifications  
|            | Measurement  | - Review and analysis of problems solved by students during the implementation of the RTM  
|            | Reflection   | - Reflections on the progress and challenges faced by students and teachers themselves  
| Third cycle | Measurement  | - Conducting the post-test to assess the impact of the RTM on the students’ achievement in word problem solving  
|            | Measurement  | - Assessment of students’ work in the post-test  
|            |              | - Joint discussion regarding the result in the post-test  
|            | Reflection   | - Teacher and research reviews relating to the joint work in the project  
|            |              | - Joint discussion on planning the inclusion of RTM in their curricula in other classes in the future  

The Intervention program: Reciprocal teaching method

As it is already known, the RTM was first developed for improving reading comprehension. It includes strategies such as *prediction, clarifying, questioning, summarizing, and planning of the solution* (Palincsar & Brown, 1984). Soon, this method was modified for mathematical word problem solving (Reilly et al., 2009; Van Garderen, 2004), and it was also used as an intervention in this study.

In order for the RTM to be used as efficiently as possible, the researcher (first author) planned a joint training with all the teachers participating in the project. The training program developed by the researchers is based on the basic features of PD proposed by Desimone (2009), that is "content focus, active learning, coherence, duration, and active participation" (p. 184). To successfully implement the project, the materials and worksheets were also prepared and discussed in joint meetings to ensure their suitability for students. The intervention was conducted by the teachers in their classes for a 12-week period with two (45 minutes) teaching hours per week.

The method requires students to engage actively in small groups so that they can discuss all strategies comprised in the method, while teachers are assigned the role of facilitator and supporter of their students. Math word problem solving process was led by the student group leader based on RTM strategies. The roles of students in groups were changed regularly in order to ensure that each student could practice prediction, exploration, questioning, and summarizing the ideas for word problem solving. From a theoretical perspective, the engagement of students in cognitive processes through RTM should be viewed from a socio-constructivist perspective. Instead of the communication of the word problem solving steps by teachers, the processes of building knowledge and skills in the classroom are considered as common social activities, where ideas and justifications are discussed by the students themselves (Tarchi & Pinto, 2016).

The whole process on planning and implementation of the RTM was led by all participants, going through the following steps:

- The clarification of strategies of RTM by the teachers themselves.
- Demonstration of strategies by the teacher and practicing them with all students.
- Assigning small groups of students, guidelines for implementing the strategies of RTM by students themselves (involving them as clarifier, questioner, reviewer, and planner).
The researchers and teachers prepared the instructions on how to use self-directed questions for their students (Meyer, 2014) for each strategy (see Figure 1).

<table>
<thead>
<tr>
<th><strong>Prediction</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Which of the information in the problem is known or related to the information you have?</td>
<td></td>
</tr>
<tr>
<td>• What is the request to be resolved?</td>
<td></td>
</tr>
<tr>
<td>• Can you draw or make a diagram to clarify how you understand the problem?</td>
<td></td>
</tr>
<tr>
<td>• Which of the information in the problem can help you the most?</td>
<td></td>
</tr>
<tr>
<td>• How will you solve the problem?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Clarification</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Which expressions or words you read are unclear?</td>
<td></td>
</tr>
<tr>
<td>• What are the unfamiliar words?</td>
<td></td>
</tr>
<tr>
<td>• What data can help you?</td>
<td></td>
</tr>
<tr>
<td>• Give examples to clarify the meaning of words and expressions.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Asking questions</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• What questions do you have?</td>
<td></td>
</tr>
<tr>
<td>• What is required in the problem?</td>
<td></td>
</tr>
<tr>
<td>• What do you think about the problem?</td>
<td></td>
</tr>
<tr>
<td>• Which information can help you?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Summary</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Tell what information you understood from the text of the problem read!</td>
<td></td>
</tr>
<tr>
<td>• What is the most important piece of the information you read?</td>
<td></td>
</tr>
<tr>
<td>• Which information can help to organize the solution of the problem?</td>
<td></td>
</tr>
<tr>
<td>• What expression summarizes the problem request?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Planning and solution</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Plan for solving the problem.</td>
<td></td>
</tr>
<tr>
<td>• Solve the problem and respond to the problem request(s).</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 1: Questions modified for RT strategies by Meyer (2014)*

During the small groups' reciprocal teaching discussions, teachers circulate among each group, listening and sometimes getting involved in the discussions, helping the students to overcome possible misunderstanding.

**Data Analysis**

Pair sample T-Test was used to analyze whether RTM had an impact on the students' achievement in solving the word problems. The statistical test was performed by using the SPSS version 23. On the other hand, in order to analyze the reflective sheets of classroom observations, as well as during joint discussions, the qualitative content analysis research methodology was used. Both researchers (authors), first explored the data details and specifics to discover important patterns and themes, and then confirming them (Cohen et al., 2007). The authors developed provisional codes based on the reflection sheets. Analysis of the codes followed, and constant comparative analyses were used to prevent research bias (Patton, 2002). Patterns and themes were then discussed by the researchers in order to explain and interpret them.

**Findings / Results**

*The impact of RTM on students’ achievement*

To examine the effect of RTM on solving word problems by fifth graders, the pre-test and post-test scores were analyzed.
Table 2 presents the basic statistics for pre-test and post-test (word problems with consistent language the solution of which requires the use of one or two arithmetic operations Gr.1 (1-5), and problems with inconsistent language whose solution requires the use of one or two, or more arithmetic operations Gr.2 (6-10). For the comparison of the results of the pre-test and the post-test, the t test is used.

Table 2. Presentation of the points in general, average scores, standard deviation in math word problems with consistent language Gr.1 (1-5), and word problems with inconsistent language Gr.2 (6-10) based on time of intervention (before/after)

<table>
<thead>
<tr>
<th>Groups</th>
<th>Pre-test</th>
<th>Post-test</th>
<th>t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td>Gr. 1 (1-5)</td>
<td>19.04</td>
<td>11.64</td>
<td>30.50</td>
</tr>
<tr>
<td>Gr.2 (6-10)</td>
<td>8.38</td>
<td>10.99</td>
<td>17.88</td>
</tr>
<tr>
<td>Total</td>
<td>27.43</td>
<td>23.63</td>
<td>48.39</td>
</tr>
</tbody>
</table>

*** p<0.001, Gr.1: (1-5) word problems with consistent language, Gr.2: (6-10) word problems with inconsistent language

The results from the paired sample t-test showed that there were significant differences between the mean scores before (M = 27.43; SD = 22.63) and after the test (M = 48.39; SD= 22.06) and t (159) = -19.667; p = 0.00, (* p <0.05).
Consequently, the p value in the 95% confidence interval was less than 0.05 (p = 0.000), which means that there was a significant difference between the pre-test and post-test means. From the data given in table 2, it is observed that students are likely to benefit from using the RTM in solving both simple, as well as more complex word problems. The effect size (Cohen’s d) for a Paired-samples T test is 0.84, which indicates the large effect of intervention.

Teachers’ reflection on benefits from PAR project

This project encouraged teachers to take a closer look at their pedagogical practices and introduce changes by transforming them into new teaching practices emerging from their collaborative, systematic and rigorous research (Wright, 2021). These changes are based on student learning practices, documenting, and reflecting on classroom problems and students’ gaps as they are involved in the learning process (McTaggart, 1997). Two themes emerged from the qualitative data analysis which highlights the teachers’ reflections on students’ benefits from RTM and their critical reflection on PAR methodological approach.

Students’ benefits from using RTM for math word problem solving

Teachers observe that RTM has an impact not only on improving the students’ skills in math word problem solving, but it also has an influence on their collaboration. They claimed that all strategies of RTM enabled students to engage actively in the problem-solving process for improving their skills and becoming successful problem solvers. Furthermore, these strategies enabled students to predict, clarify unfamiliar words and expressions, asking questions to understand in depth the problem content (Van Garderen, 2004), identify the unknowns, as well as operations which can be used for solving them. The RTM also influenced the students’ capability in summarizing the problem solutions, as well as in planning other accurate problem solving.

According to the teachers, collaboration in small groups is an important part of the process. When students engage in their roles, they seem to be more focused on reading and on choosing the right way for solving word problems. In addition, changing roles in their groups were often seen as an attractive way to engage actively in the problem-solving process.

The reciprocal teaching method has had positive effects on improving students’ ability to solve mathematical word problems. Students, through this method, managed to develop their understanding of mathematical problems, requests, and various expressions such as ‘how many times more’ or ‘how many more’. Also, the RTM strategies helped them clarify their thoughts on problem requirements. [T1]

Working in groups where everyone had a specific task was an additional responsibility for everyone and sometimes it was seen as an attractive way of learning. When many minds work on solving a problem, the solution is easier. Students were facilitators for each other. They helped each other. The students managed to understand the problems, and they were more logical when they discussed with each other in order to be clearer about choosing the right operations. [T2]

The strategies of the reciprocal teaching method are very suitable for the level of our students because they enabled them to play different roles. Being clarifier, predictor, and summarizer was not only an attractive role for each student but, at the same time, they helped them to be more correct during word problem solving. [T5]

From the teachers’ reflections, the RTM for solving word problems was especially useful and the benefits were manifold. Students were not only more careful when they read the problem and more focused on the accuracy of the
calculations, but they also had greater self-confidence to engage in the group discussions. They were involved in all phases, from clarification, which is often accompanied by the visualization, to the reasoning for the solution, and that has led them to appreciate their participation in group work (Meyer, 2014). However, for teachers it was a concern that when it comes to solving more complex problems, especially those with inconsistent language, there is a dominance of a smaller group of students in the commitment to clarify, assume or give ideas about problem solving (Boonen et al., 2016; Vula et al., 2017).

**Teachers’ benefits from PAR project**

All teachers reported in the joint meetings that there was a significant increase in students’ engagement in the group work during the word problem solving through the PAR project. The analysis of teachers’ reflections is used to argue that the PAR project has the potential to improve existing teaching practices and hence lead to transform them into more effective pedagogical teaching practices adapted for math word problem solving (Wright, 2021).

The same concern and the same approach to overcome the students’ gaps that teachers identify in their classes, enable all teachers to be more open during the communication regarding the ideas and challenges towards finding solutions. The PAR used in this project was based on equal responsibility of all researchers and teachers. Consequently, the teachers were not simply involved in the research project, but as McTaggart (1997) emphasizes, they ‘participated’ in all phases of the research project. For all the teachers, except one who had prior knowledge of conducting an individual action research, this approach of professional collaboration was a new practice and it was the first time that they engaged in such a project.

Despite the positive reflections on the PAR process, its implementation has often been challenging. The great number of administrative engagements that teachers have, often made it impossible for them to be involved more in such projects. Even working with students in the classroom sometimes poses difficulties. Hence the RTM, engaging students through all strategies, where each student is responsible for their own learning, was not easy.

Applying the RTM with students was a new experience in math classes, and in the beginning, it was challenging to organize them in small group discussions. One of the difficulties that is not easy to overcome is how to involve the students with learning difficulties in this process [T4].

Challenges were the main issues to be discussed in the joint meetings. If there were difficulties in the classroom related to the mistakes that students made, all teachers were asked to present ideas from their own experiences (Skovsmose & Borba, 2004). Sometimes it was the researchers who provided guidelines which were based on research sources. Most teachers’ reflections were focused on students’ insufficient skills for solving word problems. Therefore, their discussion was focused more on identifying the difficulties that their students have had in solving such problems. Consequently, as in many other studies, teachers mostly reflected on difficulties in understanding the text of the problems, especially those with more complex texts (Boonen et al., 2016; Pearce et al., 2013; Vula et al., 2017). The same difficulties were reflected in both word problem tests of this study.

In the joint meetings, issues such as: how the student should be guided, which role should be considered for each student, or which problem-solving strategy the student should use in order to find the solution were discussed in detail. The deep discussion on common problems, and sharing the ideas on findings, as well as the solutions for improving their students’ skills, also influenced the improvement of the teaching practices of the teachers themselves.

It was a new way of engaging us in the word problem solving process. We have rarely collaborated on such topics before. In this project, we had the opportunity to discuss in depth the issue that worried us and at the same time, we were able to find a solution. I learned that I needed to improve my teaching to solve math problems [T1].

The PAR project fostered collaboration between teachers and researchers. Working together in critical reflections, helps them to mutually support each other in addressing issues on problem solving strategies that challenge them professionally (Robutti et al., 2016). The PAR project, as teachers stated in their reflections, was different from other traditional professional development programs.

First, involvement in the PAR project was beneficial for increasing collaboration. It was different, not like before. We planned and were more organized. We also had the opportunity to collaborate with the researcher, get guidance, and clarify our ideas. For us as teachers, this collaboration was especially important because we were engaged to solve our problems [T3].

Among the benefits of the PAR project, the reflective process was considered a process from which teachers learned very much. Reflections, whether written or oral, made them think deeply about their teaching practices. Through the transformations of these practices in the classroom, they developed more empowering pedagogy (Wright, 2021), which had an impact on them and their students, too. From the joint work in the PAR project, several goals were accomplished, such as: strengthening the cooperation between teachers, students, and researchers; improving students’ learning and their problem-solving skills; developing teachers’ ability to search for new resources that may
contribute to their professional development; improving their reflective abilities, and bringing about changes in school and in their everyday work.

**Discussion**

In their study, Robutti et al. (2016) have shown that efforts to understand what teachers do for improving their teaching have led to an increased interest in exploring the different activities and the nature of various collaborations through which teachers are engaged. In this study, researchers and teachers, as participants in the PAR project, aimed to assess whether RTM affects the improvement of students’ skills and their achievements in solving math word problems. Furthermore, PAR is presented as a research design (Richter, 2016), as a collaborative and action-oriented partnership (Leavy, 2017), as a form of teacher professional development, where critical reflections and exploratory reasoning were the basis for participants’ discussions (Skovsmose & Borba, 2004). In this study, PAR is considered to be “a methodology for an alternate system of knowledge production based on the people’s role in setting the agendas, participating in the data gathering and analysis, and controlling the use of outcomes” (Reason, 1994, p. 329)

The purpose of the study was twofold: first, the impact of RTM in the students’ achievement on word problem solving was analyzed; and secondly, the focus was on teachers’ reflections during the PAR project regarding students’ work in small groups, and their reflections on their own work as participants in the research process.

It was found that the RTM was an effective method for word problem solving, especially for reading comprehension as a component of word problem solving. The results are in line with other studies, which confirmed that reciprocal teaching enables dialogue between students to examine the understanding of the text by drawing, sketching, underlining phrases in the text, or using diagrams, which facilitate the mathematical word problem solving (Kurshumlia & Vula, 2019; Meyer, 2014; Rosenshine & Meister, 1994; Tarchi & Pinto, 2016; Van Garderen, 2004). After the comparison of the results of the pre-test and the post-test, it was found that there was a noticeable improvement in the achievement of the students in solving word problems. Furthermore, the results from the class observation reflections show that the teachers considered that important factors for improving the results in the post-test were the taking of responsibility and cooperation between the students themselves. As it was shown by Tarchi and Pinto (2016), the clearly defined purpose and the use of strategies of RTM were prompting for the members of a group to help each other.

However, in both tests (before and after the intervention with RTM) the results show that students have significant difficulties when they have to solve more complex word problems and when their solution requires more than one arithmetic operation. Similarly, Vula et al. (2017) showed that although the intervention with meta-cognitive strategies had the effect of improving students' results in word problem solving, their success was much lower in problems with inconsistent language (Boonen et al., 2016).

Usually, it is expected that most teachers are ready to implement the recommendations for changes in practice arising from outside research which they know little about (Wright, 2021). In this study, teachers were more motivated to change their teaching practices. From their reflections, it is understood that PAR enables them to get involved in carrying out research themselves, regarding their own and their students' needs. Specifically, PAR allows researchers and teachers as partners to dialog into a more in-depth understanding of students’ gaps in the word problem solving process. Discussing different experiences through critical reflection, as it was stated in the study of Moreno and Rutledge (2019), allowed them much easier use of the resources and examples presented by other researchers as tools to enrich their knowledge of teaching practices. The exploratory reasoning which was involved in their reflecting on the activities tried out in their classes was especially useful. This conclusion is supported by the study of Skovsmose and Borba (2004), who confirmed that this reasoning helped teachers to better understand the practices based on RTM and to discuss and develop further ideas during the joint meetings.

**Conclusion**

One of the most important factors in solving word problems is the comprehension of the text. Therefore, proper selection of methods for reading comprehension is especially important. This research project based upon the RTM in conceptualization of the mathematics word problem solving through PAR, was focused on the involvement of students in collaborative activities, as well on the participation of their teachers in the reflections relating to both teaching and learning practices. This focus is reflected in the research findings that have implications not only for students’ outcomes and their problem-solving skills, but also in teachers’ empowerment in changing their pedagogical practices.

The results showed the increase in students’ results after their involvement in all steps of the RTM. The collaborative learning, as the main activities of RTM, enabled the development of cognitive and metacognitive strategies, and the construction and co-understanding of the text (Hartman, 1994; Tarchi & Pinto, 2016). The students’ engagement in such cognitive processes as well as in the common social activities through RTM, made them present ideas, reason, and discuss among them (Tarchi & Pinto, 2016), and be able to use self-directed questions (Meyer, 2014) as important tools for the word problems solving.
On the other hand, the important advantages of the implementation of the PAR project within the school environment helped teachers to improve their teaching practices. Teachers were not passive recipients of information about the RTM, but through action and reflection processes, they built their professional knowledge (Desimone, 2009; McTaggart, 1997). Instead of asking teachers to follow the RTM implementation steps, and then assess the impact on student outcomes, PAR has challenged them to investigate the obstacles their students face, and through searching for new sources and discussions in the joint meetings, find ways to overcome them. These joint engagements within the research project, enable teachers to transform their teaching into new pedagogical practices which are based on students' gaps as they are involved in the word problem solving process (McTaggart, 1997; Wright, 2021). The project highlighted the effectiveness of collaboration between teachers and researchers for sharing their experiences and deciding regarding the changes in their classroom practice through the PAR cycles.

**Recommendations**

The study showed that the RTM can help students improve not only reading comprehension but also a variety of skills, especially collaborative skills. Further studies should focus more on teachers in order to investigate how they apply the reciprocal teaching procedure in their classrooms. Specific attention should be paid to the student-teacher and student-student relationship in the implementation of collaborative intervention. In this way, it would be possible to explore the didactic approaches for particular concepts in greater detail, not only in mathematics, but in other subjects as well.

Furthermore, the study showed that the PAR project can empower teachers and education researchers, as well as intensify their collaborations. Consequently, the study suggests that PAR should serve as a form of school-based professional development. Moreover, this study suggests that the PAR project can be done in schools not only by teachers of the same curricular area, but also between teachers of different study fields. The same PAR project can be planned as an internship of teachers within the school, or as a form of even wider collaboration with teachers from other schools. In addition, teachers’ reflections should be part of the professional practices. Therefore, further studies can be conducted aiming to explore other potential benefits of using the RTM and PAR methodology.

**Acknowledgments**

The authors would like to thank the teachers who participated in the study.

**Authorship Contribution Statement**

Kurshumlia: Conceptualization, design, data gathering, data analysis and interpretation, drafting manuscript. Vula: Conceptualization, data analysis and interpretation, editing/reviewing, supervision.

**References**


