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Developing Mathematical Communication Skills for Students in Grade 8 in Teaching Congruent Triangle Topics

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Abstract: Teaching mathematics in general and instructing mathematics at junior schools in particular not only create favorable conditions for students to develop essential and core competencies but also help students enhance mathematical competencies as a foundation for a good study of the subject and promote essential skills for society, in which mathematical communication skill is an important one. This study aimed to train students in mathematics communication by presenting them with topics in line with the structure's congruent triangles. An experimental sample of 40 students in grade 8 at a junior school in Vietnam, in which they were engaged in learning with activities oriented to increase mathematical communication. A research design employing a pre-test, an intervention, and a post-test was implemented to evaluate such a teaching methodology's effectiveness. For assessing how well the students had progressed in mathematical language activities, the gathered data were analyzed quantitatively and qualitatively. Empirical results showed that most students experienced a significant improvement in their mathematical communication skills associated with congruent triangles. Additionally, there were some significant implications and recommendations that were drawn from the research results.

Keywords: *Congruent triangles, mathematics education, mathematical communication skills, the teaching process.*

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Introduction

In mathematics classes, many forms of communication can take place. This activity can happen through interaction with the teacher, small group work, or standing before the class to present a presentation to clarify a found idea. Teachers can let students face and discuss, encouraging them to speak up their ideas and take time to discuss with people around them; this is especially beneficial for students who are less confident when sharing ideas in front of the class. It is believed that communication is an essential part of mathematics and mathematics education. A way to see communication is to perceive it as sharing ideas, being open to discussing these ideas, and reflecting on them quickly (Makur, 2019). When students communicate mathematical ideas, they are strengthening their understanding of mathematics.

Teachers must be the only connection through which information may be passed in the teaching process. Teachers use various means of delivering information to students during the lecture, including language, speech, writing, and other audiovisual devices. Since students are recipients, they can easily hear, deduce, evaluate, and change the information in messages based on their knowledge and experience. After receiving the message, students will have reactions or feedback: taking notes, listening, commenting, answering verbally, writing or attitudes (surprised, confused, or disagree). There is an undeniable need for feedback in communication; it aids the source in identifying, correcting, or revising the message to better match reality. The author also emphasized that the communication process is dynamic, unstable, and always changing; the communication flow factors are always interactive. Communication is broken down into steps in teaching, and in that way, interactivity and contact make the communication process transparent.

Mathematics is a suitable subject to develop communication because mathematical communication and mathematical thinking are necessary for future life; mathematics is a unique language consisting of words, tables, drawings, graphs, and symbols. When students are challenged to think, explore and explain a math problem, present the results by writing or speaking, and arguing, their knowledge will be steady, and their learning will be more effective. We acquired

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two outcomes during that time: students use communication to help them learn math, and students employ communication to aid their learning. When evaluating a student's capability to solve problems (Rohid et al., 2019) and create conditions for their students to utilize their ability to speak about mathematics, teachers have something to base their evaluation on.

It is proved that mathematical representation, explanation, argument, and presentation are related to mathematical communication. Students can use mathematical representations to express their views and ideas through words or writing on paper. In this case, the explanation is a method by which students can better understand their ideas and viewpoints by discussing with others. Teachers enable students to exercise their critical thinking skills by reasoning with them. Finally, students present proof so that others understand the problem clearly and accurately. The basic mathematical communication methods include mathematical representations, explanations and arguments, and presentations that motivate students to share, exchange, and reflect in the learning process (Rahmi et al., 2017). In the process of explaining, arguing, debating, issues will appear when students work in groups or they have to exchange ideas with their friends. Smieskova (2017) stated that mathematical communication skills are considered tools to develop students' creativity and motivation.

Mathematical representation describes relationships between objects and symbols, a bridge to communicate with others easily, be it signs on paper, drawings, sketches, graphs, charts, geometric outlines, and equations. Students will increase their debate on issues while demonstrating their understanding of math concepts related to problem-solving (Sari & Darhim, 2020), the explanation and discussion stages. Students discover and learn various methods to demonstrate their knowledge and determine whether the statement is correct. Conventionally, it is assumed that the argument is a systematic arrangement of arguments that ultimately solve a problem (Shahbari & Daher, 2020). Note that one can argue with absolutely no regard for the truthfulness of the conclusion that one would like the listener to approve. Students can prove or disprove counter-examples, right or wrong (Salsabila, 2019). Thus, it follows that the argument refers to discovering mathematical proofs. When it comes to the expression 'proving the theorem or the truthfulness of a particular judgment,' providing proof is one of the students' writing or verbal expressions that could be employed to argue a theorem or the veracity of a specific judgment in order to assist others to understand the problem. Teachers assign four primary forms of classroom communication in mathematics: verbal communication, listening, verbal communication via reading, and written communication (Utomo & Syarifah, 2021; Wilson, 2009).

It is crucial to monitor some standards of mathematical communication. Students first organize and formulate ideas using a variety of representations. Second, they can present their thoughts (point of view) coherently and clearly with their peers and teachers. The third way learners can solve math problems for their peers is to examine, evaluate, think, and even solve math problems for other students. Finally, students can use their mathematical abilities to express ideas accurately.

It is likely that during math communication, students will encounter several obstacles. The forms of mathematical communication in math classrooms all occur in math classrooms to solve math problems (Rohid et al., 2019), helping students comprehend mathematical knowledge and skills. At that time, students must listen, debate, explain and prove math problems logically and accurately. To do this, first of all, students must have good mathematical knowledge. In other words, not good mathematical knowledge is a big obstacle in the mathematical communication of students. Next, the mathematical vocabulary is an essential means in mathematical communication; without the mathematical vocabulary, the mathematical communication process cannot occur. With the demands placed on writing skills, mathematical vocabulary limitation should not be a barrier in mathematics communication.

Additionally, students' lack of confidence in communicating is also an obstacle in mathematical communication. These students often dare not speak or ask questions for teachers and are afraid to argue with other students about the lessons' content, so they almost do not participate in verbal communication activities in the classroom. Moreover, critical thinking plays a fundamental role in making decisions; it helps teachers build up the right questions (Nuraina & Mursalin, 2018), evaluate possible answers, and assess sources' reliability. Therefore, if students' critical thinking is still low when participating in mathematical communication, they will not have the habit of considering whether the information given by the teacher is accurate and easy to accept the problems. The problem is not understood or adequately explained. Since then, the mathematical communication process between teachers and students will take place one way and not achieve high efficiency.

To influence math communication skills, teachers can use lesson plans that are educationally oriented in Realistic Mathematics Education (RME) (Andriani & Fauzan, 2019; Arnawa & Ismail, 2020; Hasibuan & Amry, 2017; Hutapea et al., 2019; Indah Nartani et al., 2015; Rahman et al., 2012; Supriyanto et al., 2020; Trisnawati et al., 2018; Widada et al., 2018). Additionally, the authors have pointed out differences in the effects of problem-based learning (Hidayati et al., 2020), RME, and inquiry learning (Hasibuan & Amry, 2017); another study is based on project-based learning models with scaffolding (Paruntu et al., 2018). Meanwhile, some other authors want to increase students' mathematical communication skills thanks to learning models such as team quiz (Johar et al., 2018), location of school (Juliarta & Landong, 2020), PQ4R strategy (Makur, 2019), assimilation and accommodation framework (Netti et al., 2019), a question on Pythagoras (Nuraina & Mursalin, 2018), digital teaching module (Setiyani et al., 2020), ASSURE learning design (Sundayana et al., 2017), the brain-based learning approach using autograph (Triana et al., 2019), the Treffinger teaching model (Alhaddad et al., 2015) and computer-supported reciprocal peer tutoring (Yang et al., 2016). There have been many studies on mathematical communication skills about many mathematical topics such as algebra problems (Paridjo & Waluya, 2017), relation and function (Setiyani et al., 2020), and algebraic factorization (Disasmitowati & Utami, 2017).

In the study, Alexander (2018) investigated, compared, and described students' achievements and improved their mathematical communication skills. It was based on the previous mathematical knowledge (high, medium, and low) using the Treffinger model and typical learning style. This study was an empirical study with all Faculty of Mathematics Pedagogy students who have studied discrete Mathematics from a university in Ternate City. Within the Treffinger and common learning framework, there were no interactive effects between learning and previous math knowledge to improve students' mathematical communication skills (Minarti & Wahyudin, 2019).

There has been much research attached to math instructional skills in math classes. For example, the authors described students' written mathematical communication skills in open math problems based on their mathematical abilities. By using descriptive qualitative research, the educators were able to expand on the situation and make recommendations. The tools used are math proficiency tests, written math communication skills tests on open-ended problems (Zayyadi & Saleh, 2020), and interview guides. The results showed that the subjects with high and average mathematical ability could satisfy three written communication skills indicators. That was the ability to express mathematical ideas through writing and performing. The task of conveying mathematical ideas and relations to written situation models in an understandable manner involves understanding, interpreting, and evaluating written mathematical ideas and other visual forms and also uses written mathematical symbols in the structure of written situation models.

Besides, the better domain-independent cognitive style depends on the problem-posing model with real-world math education as useful as the problem-posing model but more effective than the direct-instruction model and the problem-posing model (Kamid et al., 2020). According to each cognitive style and the problem-posing learning model with the Indonesian real-world math education approach, the field-independent cognitive style is the same as the dependent school but independent of the school. Fields that are more field-dependent are seen in instructional learning models and can be better for learning.

Additionally, there was also an impressive result on the gender difference and the students' school background in the mathematical communication skills in the research of Hayati et al. (2020). Compared to their male counterparts, female students had an advantage concerning specific abilities. Students in public elementary schools were more likely than those in private elementary schools. Moreover, the students' ability to draw was higher in writing and the mathematical expression aspect. Another study also aimed to boost students' mathematical communication skills using the CORE learning model by Yaniawati et al. (2019). The idea that students' mathematical communication and connections were best instead of expository learning was reported as more efficient. On the other hand, there was a specific relationship between math communication, mathematical connection (Minarti & Wahyudin, 2019; Sari & Darhim, 2020), and mathematical disposition.

Some studies in the world were attached to the topic of congruent triangles such as SAS and SSA conditions for congruent triangles (Alexander, 2018), the conceptual explanation of congruence and similarity in the real world context and conceptual knowledge (Dündar & Gündüz, 2017), the comparison of geometrical features of two congruent triangles (Leung et al., 2014), and student's understanding of the relation between the two different kinds of congruent triangles that appear in a dynamic multi-touch geometry environment (Yenny, 2016). In particular, in their research, the authors, Shahbari and Daher (2020), studied the effect of using ethnography in Islamic contexts on learning the topic of congruent triangles. Thirty 10th graders took part in ethnomathematics by learning about congruent triangles using Islamic decorations to gain this goal. Before the study, students were asked to answer one questionnaire, and after that, they were given two more. The main results indicated that students successfully constructed concepts of congruence and congruent triangles through the ethnomathematics process. Thus, the students succeeded in finding and constructing three congruent theorems. Furthermore, the findings related to the questionnaire showed that the students' ability to back up their evidence improved due to ethnomathematics-based learning.

Another study was about the errors of 8th-grade students when they made reasoning and proof on the topic of congruent triangles in China (Wang & Wang, 2018). The authors investigated what Chinese 8th-grade students made in their arguments and proved that the triangle was congruent. There were 102 participants, both male, and female, in two grade 8 middle schools in China. The results showed that they were confused with the five theorems' connotation and form about the congruent triangle. Students had created standard graphs, but it was difficult for them to use them in visual analysis. Also, they had difficulty switching between representations, mathematical language, natural language, and figures. Not only were the students unable to devise an exact proof process, but also they had difficulty with approximating it. Additionally, the authors also developed a scale to evaluate students' ability to reason and prove when learning the topic of congruent triangles. A statement from researchers found that graduate students made approximately half of the errors discovered in the examination of Wood (2012), and all of the errors discovered by them in the study appear.

Theoretical Background

Mathematical communication skill, congruent triangles in curriculum and mathematics textbooks in Vietnam

According to the General Education Curriculum (Vietnamese Ministry of Education and Training, 2018), math teaching is oriented to form ten core competencies. Specifically, the competencies include three general competencies: autonomy and self-study, communication and cooperation, problem solving and creativity, and seven professional competencies that follow the subject system in each grade level. Accordingly, the competencies need to be formed and developed for learners through teaching Mathematics in high schools, including thinking capacity and mathematical reasoning; competency in mathematical modeling; ability to solve math problems; mathematical communication competency; ability to use mathematical tools and means. Mathematical communication skills related to the effective use of mathematical language (letters, symbols, diagrams, graphs, logical connections) combined with a common language; this ability is shown through mathematical texts, asking questions, answering reasoning questions when proving the correctness of propositions, solving math problems.

In other words, mathematics communication skills entail knowledge, skills, and attitude. The fact that students must have mathematical knowledge and expertise are well known. Following that, they have learned how to use mathematical language (words, terms), accomplish, and make their ideas more understandable. Finally, students must have a spirit of cooperation, sharing, exchange, and positivity on mathematics issues. Mathematical communication skills include the following elements:

(1) Listening to understanding, reading comprehension, and taking notes of necessary mathematical information presented in mathematical text or spoken or written by others.
(2) Presenting, expressing (speaking or writing) mathematical contents, ideas, and solutions in interaction with others (with appropriate completeness and accuracy).
(3) Effectively using mathematical language (numbers, letters, symbols, charts, graphs, logical connections) combined with a common language or physical movements when presenting, solving like, and evaluating math ideas in interactions (discussing, debating) with others.
(4) Showing confidence when presenting, expressing, asking questions, discussing, and debating mathematics-related ideas.

Thus, according to this program's point of view, mathematical communication skills integrate two components: representation capacity and communication capacity. These are two essential elements that work in tandem; they are supportive of and bolster each other. From the above points of view, mathematical communication skills include mathematical knowledge, skills to use mathematical vocabulary (the language of mathematics), representational forms of mathematics, and the ability to express and explain understandable ideas for others to take in.

In this program, the specific manifestations of mathematical communication skills and requirements to be met for junior high school students are as follows. Firstly, students listen to comprehension, read comprehensively, and take notes (summarizing) the necessary math information, the text's focus (in the form of written or spoken text). From there, students analyze, select, extract necessary mathematical information from the text (in the form of written or spoken text). Next, they perform, express, question, discuss mathematical contents, ideas, and solutions in interaction with others (at a relatively complete and accurate level). Also, they use mathematical language combined with a common language to express mathematical contents and show evidence, methods, and argument results. From there, they show confidence when presenting, expressing, discussing, arguing, explaining mathematical contents in some non-complicated situations.

Considering the content of the topic "Congruent Triangles" in Vietnam's curriculum and textbook (Chinh et al., 2007), the program sets the goal of the topic "Congruent Triangles" according to the following skill knowledge standards:

(1) About knowledge: understand the definition of two congruent triangles; understand the theorems (the congruent cases of two triangles, the congruent cases of the right triangle).
(2) About skills: apply congruent triangular theorems to solve mathematical problems (Prove that two triangles are congruent, find the length of the line, calculate a constant ratio, the ratio of the perimeter, ratio of area, calculate the perimeter, area of the triangle, prove equalities); know the application of congruent triangles to solve real-world problems.

Starting the topic "Congruent Triangles" is a concept lesson of two congruent triangles. Before giving the concept of two congruent triangles, the textbook begins the lesson with the question: "What are two congruent triangles?", This is also a problem that needs to be resolved after finishing the lesson. Next, the textbook gives pairs of shapes with the same shape (see Figure 1) but different sizes and concludes that such pairs are called congruence (Dündar & Gündüz, 2017).

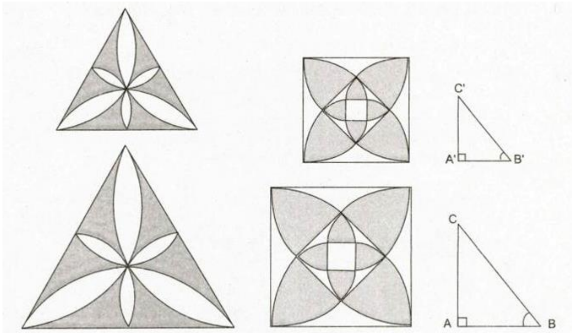

Figure 1. Examples of congruence pairs in textbooks

After that activity, the textbook defines two congruent triangles as follows: A triangle ABC is said to be congruent to a triangle $A'B'C'$ if: $\widehat{A'} = \widehat{A}$; $\widehat{B'} = \widehat{B}$; $\widehat{C'} = \hat{C}$; $\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'A'}{CA}$.

The triangle ABC is congruent with the triangle $A'B'C'$ symbolized $\Delta ABC \backsim \Delta A'B'C'$ (written in the order of the corresponding pair of vertices). The ratio of the respective sides $\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'A'}{CA} = k$ is called the congruent ratio.

The congruent cases of the triangle are presented in the textbook as follows:

(1) If the three sides of this triangle are proportional to the other three sides, then those two triangles are congruent.
(2) If the sides of this triangle are proportional to the two sides of the other and the angles made up by the pair of sides are equal, then those two triangles are congruent.
(3) If the two angles of this triangle are equal to the other two, then those two triangles are congruent.

Next, the textbook presents the homologous cases of the right triangle from the application of the congruent cases of the triangle as follows: Two right triangles are congruent if: This right triangle has an acute angle equal to that of the other right triangle;

This right triangle has two right-angled sides proportional to the other two right-angled sides (Alexander, 2018).

Furthermore, the textbook also presents theorems related to the ratio of two high lines, the ratio of the area of two congruent triangles. The ratio of the two corresponding altitudes of the two congruent triangles is equal to the congruent ratio. The ratio of the area of two congruent triangles is equal to the square of the congruent ratio.

Regarding practical applications of congruent triangles, the textbook presents two typical problems: Indirectly measuring the height of an object and measuring the distance between two locations, including an inaccessible place.

The exercises featured here use a wide range of problems to demonstrate that textbooks are flawed in helping students build their ability to communicate mathematical concepts effectively. For example, the form of math "Applying congruent triangles to real-world problems" is a problem that requires students to satisfy the criteria of the mathematical communication competence such as vocabulary mathematics (perpendicular, plane), mathematical representation (from natural language to mathematical language to represent on specific drawings), explanation (understanding the problem requirements, reasoning and presented reasonably). However, such problems appear quite a few in textbooks (2 out of 33 exercises on the topic of congruent triangles). Although an essential component of textbook definitions is integrating mathematical vocabulary explanations, creating textbooks is an educational process that leads to that goal.

According to the research team and some other teachers' experience, the topic of "Congruent Triangles" is not tricky knowledge, but they are confused about presenting the solution verbally and on the board. Some students indicated that they comprehended the lesson and comprehended the concept, but they were unsure how to present it. Understand the students' difficulties, but the teacher added that it had not mentioned much about communication in teaching in the practice of training and retraining teachers today. Many teachers have not had specific measures to organize students to participate in learning activities and mathematical communication dynamics. Therefore, developing mathematical communication skills is vital, creating a foundation to stimulate their learning ability, helping students be confident and interested in learning, and contributing to clarification and adding innovative teaching orientations to develop students' mathematical competencies. Moreover, it enhances students' accountability and activeness, the initiative in lesson building, creating solid knowledge of themselves, forming and developing the ability to connect mathematics with practice.

Evaluating mathematical communication skills

When it comes to the math teaching capacity of junior high school students, we are interested in the ability to understand, receive and comprehend the math content that is spoken and written; the ability to create meaningful mathematical messages; ability to present coherent, accurate, logical, confident and convincing when expressing their mathematical views in exchanges and discussions. This scale intends to ascertain one's aptitude to accomplish a specific task, shown in Table 1.

Table 1. Criteria for evaluating mathematical communication skills according to 4 levels

Levels Criteria	Level 1	Level 2	Level 3	Level 4
Math vocabulary (Words, terms)	Using incorrect or not using mathematical language.	Using mathematical language, but still has many errors.	Using mathematical language is almost correct.	Using mathematical language correctly, without errors.
Mathematical representation (Symbols, number axis, models, drawings, graphs, charts)	Performing incorrectly or fails to represent the problem as required.	Correctly representing part of the content of the problem.	Performing precisely the kind of representation required by the problem.	Using multiple representations precisely as required by the problem.
Explain (Argument, presentation)	Lack of grounds in the argument, presentation, right or wrong answer but no explanation.	There are explanations, but omitting some details, the answer may or may not be correct.	Has explained all the details and got the correct answer.	Clear, logical, and coherent explanations include all details with correct answers and generalizations and extensions.

Methodology

Research Goal and Questions

The fact shows that the congruent triangle topic is good for forming and developing students' mathematical communication skills. Therefore, this study aims to form and enhance these skills for 8th-grade students by teaching congruent triangle topics. In order to fulfill this goal, research is used to answer the following questions:

1. What do students learn about congruence and two congruent triangles?

2. When students learn about congruent triangles in a four-phase teaching process, how will they improve their mathematical knowledge?

3. How did students develop their mathematical communication skills after they learned from the process above? Do they have any difficulty in communicating mathematically?

Sample and Data Collection

This study's experimental sample included 40 students of grade 8A1, Quan Co Thanh Secondary School, Chau Thanh District, An Giang Province, Vietnam, and approximately 14. These students did not learn anything about two congruent triangles. These participants were selected because they were both available and willing to participate in the learning process. This research also indicated that neither prejudice nor disrespect for students was being studied, and it found no adverse effects on them.

In order to ascertain the goal, some tasks were set out. First, the researchers designed pre-test and post-test on an experimental class. The lesson plan for the next step in this study follows, in which experimental methods are used to enhance students' mathematical communication skills. The research team conducts teaching, observing, and collecting information reflecting the experimental process related to the teaching process's feasibility and effectiveness. After that, the analysis and processing of experimental data by the statistical method of evaluating students' mathematical communication ability are performed, thereby evaluating the experimental results. As a result, data were compiled from several minutes of instruction and the worksheets students filled out.

It was decided that evaluating teaching effectiveness by designing a single group's pre-test and post-test would be fitting. The design of this study was thoroughly methodologically commented on by Marsden and Torgerson (2012). Here was how to describe the design on a single group.

Table 2. The study design process

Pre-test	Intervention	Post-test
O_1	X	O_2

In Table 2, before the experiment, we conducted a preliminary test on a selected group of students to see if our measurement approach was appropriate. We would then have the students who participated in the previous experiments take the post-test for the same group. The results of the post-test and pre-test exams were used to conduct the preliminary research to determine whether or not the hypothesis was correct. To determine if the experiment would be successful, it was important first to validate and test the learning and research instruments. The researchers implemented the same grading process for the Vietnamese national exam (see Figure 2).

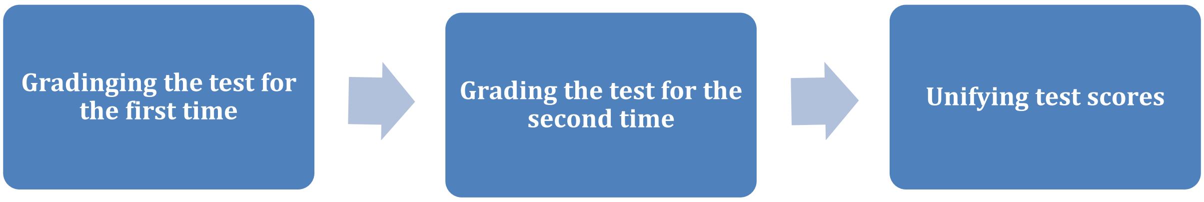

Figure 2. The grading process for the tests

(1) When grading the first time, apart from the slashes on the remaining blank pieces of paper on the candidate's test paper, the examiner does not write anything on the candidate's test and the test bag; partial scores, overall scores and comments (if any) are recorded in only one scorecard of each test; on the scorecard clearly state the full name and signature of the examiner.
(2) After grading the first time, a teacher pulled out all the test scores; then give the test bag to the teacher for the second time. The second time examiner records the marks directly on the student's test (the score for each small point must be written in the left margin of the test next to the marked idea) and on the score sheet.
(3) During this period, two examiners discuss and agree on the score; the second time marking officer scores points; two examiners jointly sign and write their names on all test papers of candidates. One of the requirements is that the two examiners must agree on each component score, then the whole test score. If the two examiners do not agree on the score, another teacher will grade the test a third time.

The experimental process

In this experiment, the experimental process was completed in five distinct steps.

(1) Step 1: Preparing for the experiment.

Develop a detailed plan for the experiment and then identify the purpose, object, content, process, and experimenting method; Select and design lessons to organize experiments; choose an experimental class; meet and exchange with teachers of mathematics in experimental class to understand the experimental period's intentions; study documents, equip knowledge and skills necessary for the experiment process.

(2) Step 2: Asking students do the test before the intervention.

(3) Step 3: Conducting experimental teaching and using some measures to influence students to develop mathematical communication skills by teaching the concept of two congruent triangles.

(4) Step 4: Asking students do the test after the intervention.

(5) Step 5: Evaluating the experimental results through discussions with the mathematics teachers about the study's issues.

Organize the experimental class to check after the experiment, analyze the obtained results, process the test data; synthesize and evaluate the measures' effectiveness to develop students' mathematical communication skills.

Analyzing the experimental lesson plan

In the current math textbook (Chinh et al., 2007), the topic "Congruent Triangles" belongs to Chapter III: Congruent Triangles. In the chapter, the readers will be given information about the concept of two congruent triangles and how to prove two triangles are congruent instances.

In this lesson plan, the teacher introduces the concept of two congruent triangles and exercises to reinforce this content towards training mathematical communication skills, requiring students to grasp math vocabulary. Mathematical representation, explanation related to the concept of two congruent triangles. As follows:

(1) About math vocabulary (words, terms): congruent triangles, congruent ratio, the corresponding angles, pair of edges corresponding to the ratio.
(2) About mathematical representation: triangular representation and congruent ratios.
(3) Regarding explanation (argument, presentation): arguments and presentations prove that the two triangles are congruent, use "written" language to reason and present ideas, use "speaking" language to express ideas.

A 4-phase teaching process was used to develop mathematical communication skills for students in teaching periods of new knowledge and exercises, incorporating mind maps to review learned knowledge. Teaching was divided into four distinct phases, which include:

(1) Phase 1: Personal work. Students are assigned by the teacher with an unfamiliar task, the task that students must think about and perform that task.
(2) Phase 2: Group work. Students work in groups with the same task in phase 1. In this phase, students choose the answers through discussion among group members. Working results of each group can be put into posters for teacher evaluation.
(3) Phase 3: Debate. The teacher chooses a group poster (usually the group with the wrong answers) to discuss. In this phase, teachers need to let students develop their ability to reason.
(4) Phase 4: Institutionalization process. Teacher comments and explains the problem based on the results of the group work of students.

Analyzing pre-test

Problem 1: Calculate x such that the area of a rectangle $ABCD$ is three times the triangle area ADE. Write an assumption, requirement for the problem and present your solution (see Figure 3).

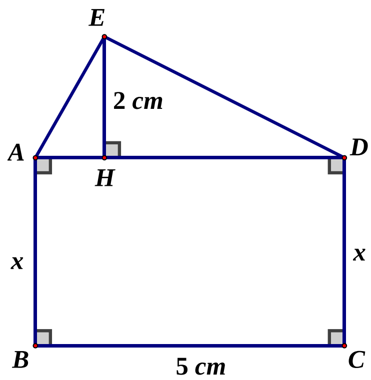

Figure 3. Illustration for problem 1

This test is constructed with the desire to be able to evaluate:

(1) Ability to read and understand mathematical symbols, formulas, and mathematical expressions.
(2) From the problem, students can write assumptions and conclusions for the problem.
(3) Ability to reason and present solutions.

Analyzing of data

This study employed various instructional materials and learning approaches, such as student worksheets, lesson plans, and communication tests that assessed mathematical skills to gather data and information, including the types of activities in which teachers and students participated. The data used in this study came from a test instrument that measured the students' mathematical communication skills before and after being exposed to the treatment. In developing the procedures and tests, researchers based on developing correct and high-quality instruments undertook the effort. When it came to the reliability of the tests, we spoke to two experts in the mathematics education field who felt they were credible; this approach was also made similarly in the research of Salsabila (2019). After consulting with experts and evaluating their suggestions, the researchers incorporated those modifications into the instruments and research they conducted. In the case of a controversy regarding the accuracy of an instrument, a panel of experts would examine the situation and verify that no more revisions were made to the instrument, and each expert asserts that the instrument was appropriate. They ultimately concluded that the tests were relevant to the research topic, so they agreed to review them. Additionally, the authors, Thao et al. (2020), suggest that one can also judge the reliability of a test by its coverage of academic content and skills; this way was used to design the test in this study. To provide a more precise answer, in this study, the tests were constructed to measure a student's ability to communicate mathematics in various forms, including the use of mathematical terminology, mathematical representation, and presentation of ideas and concepts to solve a specific math problem associated with two congruent triangles.

Concerning the findings of this study, the tests here were all designed around the test's content validity, which was used in the studies of Thao et al. (2020) and Salsabila (2019). Content that was previously selected before the pre-test was related to the triangle area and the rectangle area. This set of contents was closely related to the textbook lessons on two congruent triangles as well. Meanwhile, the post-test was a problem about two congruent triangles such as "On one side of angle xOy (xOy) ≠180°), place the line segments *OA=5cm*, *OB=16cm*. On the second edge of that angle, place the line segments *OC=8cm*, *OD=10cm*. Prove that two triangles are congruent". This problem was designed to assess students' ability to read and understand mathematical symbols, formulas, mathematical representation, and express problem-solving ideas through its solutions. Besides, a detailed rating scale was also used for two separate tests measuring a student's capacity to comprehend mathematical content, such as learning new terms in math, representing ideas using mathematics, and interpreting it.

The teaching period minutes were analyzed qualitatively to assess students' perceptions of three aspects: knowledge, skills, and attitudes. Meanwhile, the worksheets in the hands of students were evaluated both quantitatively and qualitatively. To determine how reliable the data was, we used a statistical test. Specifically, data were analyzed using the paired sample t-test with the help of the IBM SPSS Statistics 25 program. Moreover, each student's worksheet involved with both pre-test and post-test was scored on a 10-point scale, then the class scores are tested on a t-test to evaluate the effectiveness of the four-phase teaching process to evaluate its efficiency according to process pre-test, intervention, and post-test. Besides, these worksheets were also graded on a four-level scale outlined in the theoretical basis to evaluate students' mathematical communication skills through the teaching process. Finally, some students' specific worksheets used qualitative analysis to assess the above skills, thereby pointing out some of the students' advantages and limitations, even difficulties in solving mathematical problems related to congruent triangles.

Results

Results related to pre-test and post-test in a quantitative aspect

After a set of test scores were given out to class 8A1, the data was recorded and seen in Table 3, where they recorded statistics on scores before and after the experiment.

Based on these results, can the researchers conclude that grade 8A1 has better learning results than when they have not affected after the intervention? For testing this hypothesis, a t-test of a paired sample was conducted. Let X be the scores of the 8A1 class before the experiment, μ_X be the average score of the students in grade 8A1 before the experiment. After the experiment, let Y be the grade of the 8A1 class, μ_Y be the average score of this class.

Hypothesis selection: $H_0 : \mu_X = \mu_Y$ (i.e., there is no appreciable difference in the average score of the experimental class between pre-test and post-test).

Counter hypothesis: $H_1 : \mu_Y \neq \mu_X$ (that is, there is an appreciable difference in the average score of the experimental class between pre-test and post-test).

Table 3. The results of paired samples statistics

	Mean	N	Std. Deviation	Std. Error Mean
Pre-test	5.7	40	1.6517	0.2612
Post-test	6.55	40	1.5517	0.2453

In Table 3, the mean of the class with the 4-step learning process was higher than its mean before the experiment. It can be concluded that the math learning outcomes for the class after the experiment were better than before the experiment. By expanding the idea of learning the process above, after the students used this approach, their math communication skills were more dispersed than when using the normal process. Before and after the intervention, the mean values of the tests indicated a significant change following the students' analysis with the above learning model. To illustrate this point, consider that the experimental class students showed improvement in their ability to communicate mathematically.

Table 4. The results of paired samples test

		t	df	Sig. (2-tailed)
Pair 1	PRE-POS	-11.448	39	0

The significance level of 0.05 used in Table 4 led to a sig value of 0.000 < 0.05, which was found. As the researchers concluded, the value to which the hypothesis applied was located in the rejection domain, and the researchers, therefore, concluded that the hypothesis H_0 was rejected and accepted H_1 instead. In other words, there was an adequate difference in the average score of the experimental class between pre-test and post-test. This outcome concluded that students' learning efficiency in grade 8A1 after the experiment was higher than before the experiment. If inferred, the students' skills to communicate mathematics after the intervention was promoted.

Results related to post-test in a qualitative aspect

In order to have more information reflecting the effectiveness of the teaching process, researchers made pedagogical observations through timekeeping, coordinated quick interviews with teachers and students, studies were broken notes, test results, check the impact of students and after the impact of students, observe the results of the expression of math communication skills of students after experimental class of students in the experimental class.

When starting the experimental process, it was documented that students in the experimental class had the following symptoms. The students had a sense of learning, listening attentively, incredibly in-class time, and performing tasks given by the teacher, such as answering questions, making presentations, performing group activities, and participating in lesson development. Nonetheless, by comparing them in groups, it was proven that there were some limitations, as follows. Students would often misuse difficult-to-understand words or employ nonsensical phrases like natural language and math when talking to each other. It was entirely arbitrary for some students to use notebooks for note-taking; titles were the only thing they could write down. Study results showed that some difficulties were experienced in the study of Shahbari and Daher (2020).

While it was confirmed that most students struggled to speak in front of their peers, this was due to a lack of confidence, an inability to speak confidently, and inappropriate language use. Many students were capable of understanding math problems but did not feel comfortable using mathematical language. Students were close to having many problems when it came to sharing their thoughts and communicating their understanding. Therefore, the teacher needed to help students overcome psychological obstacles such as fear of being wrong, ridiculed, or afraid to express their views in front of their classmates to participate in communication and effective communication actively. In terms of delivering information, neither the speed of information transmission nor its accuracy was satisfactory. Because of this, they would usually have to ask their friends to find out about the teacher's request, or the teacher would have to repeat it multiple times. Additionally, students' notes were often haphazard, careless, or slow to take notes, and they occasionally failed to take notes accurately. Concerning the level of mathematical communication skills that could be tested via two tests before and after the intervention, the results can be found in Table 5:

Table 5. Sample results matching the level of mathematical communication skills

Criterion	Levels	Level 1	Level 2	Level 3	Level 4
Math vocabulary	Pre-test	4	12	17	7
(Words, terms)	Post-test	0	0	29	11
Mathematical representation	Pre-test	1	5	24	10
(Symbols, number axis, models, drawings, graphs, charts)	Post-test	0	2	28	10
Explain	Pre-test	7	13	18	2
(Argument, presentation)	Post-test	5	6	23	6

Although students were familiar with summarizing the problem by finding assumptions and conclusions from grade 7 when they received the test, they appeared bewildered and did not know what to do. Some students also expressed their questions to the teachers: "What is the assumption, the conclusion?". The result was that some students understood the requirements but did not know how to go from the problem requirements to the symbolic representation. Also, as a result, the lesson's content was presented using the summary format.

Students had problems writing assumptions and conclusions and could not present the problem despite their efforts; Student07 was a common occurrence (see Figure 4). Once it was observed that this was not the student who removed the blank paper and did not want to do it, it became apparent that he had worked hard enough to prevent him from submitting a solution for the problem yet, which meant he was not yet able to write the assumption and conclusion for the problem.

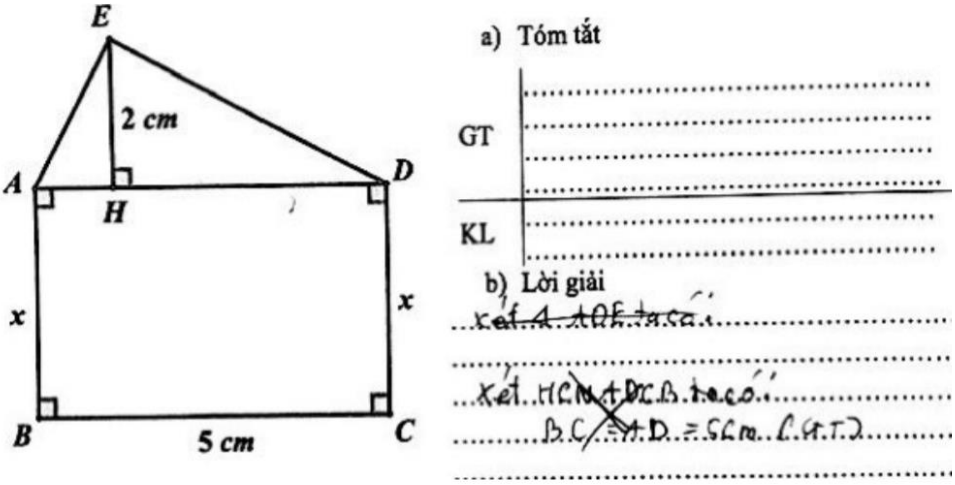

Figure 4. Student07's worksheet

(Translate in Figure 4: a) Summary: Given/Prove; b) Solution: Consider the triangle *ADE*, we have: Consider the rectangle *ADCB*, we have: *BC* = *AD* = 5cm (from the asumption))

In the meantime, Student12 presented the solution but could not write assumptions and conclusions of the problem, which led to this student's reasoning and presentation, which led to results that were not the problem's requirement (see Figure 5). This representation was a common occurrence for many students; they had difficulty comprehending the problem requirements and frequently stopped calculating a rectangle area. Although at the beginning of the lesson, it clearly stated: "Calculate *x* so that ..." but stopping at the result of the area of a rectangle or triangle area accounted for a high proportion of the total number of students' problems.

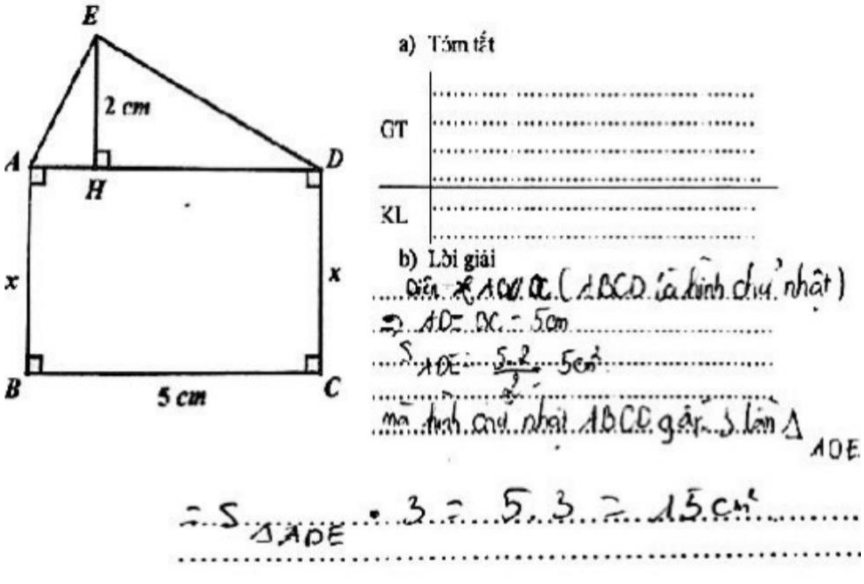

Figure 5. Student12's worksheet

(Translate in Figure 5: a) Summary: Given/Prove b) Solution: Consider the rectangle *ABCD*, we have: *BC* = *AD* = 5cm, $S_{ADE} = \frac{5\times 2}{2} = 5\ (cm^2)$. Since the rectangle *ABCD* is three times the triangle *ADE*, we have: $S_{ABCD} = S_{ADE} \times 3 = 5\times 3 = 15\ (cm^2)$).

However, many students entirely solved the problem but were still able to write assumptions and conclusions of the problem, from which it was seen that the number of students using mathematical symbols, understanding the mathematical text of the problem was still minimal. Additionally, in the pre-test, one student had the maximum score, but only 1/40 of the total number of students accounted for 2.5%.

Most of the post-experimental tests were obtained with very positive results; several students did not solve the problem by average results but wrote the assumption to conclude and solve the math problem, despite many shortcomings. For instance, Student05 made remarkable progress from not writing assumptions and conclusions. This student demonstrated a complete understanding of the concept she set out to prove, concluding that she must have assumed the problem. This outcome is consistent with the statements that Yenny (2016) has made. While some steps toward mathematical communication had been taken, a positive signal showed further progress.

A significant gain was made in the number of students who completed the exercises, including formulating assumptions, drawing conclusions, and solving problems (Rohid et al., 2019). Despite several students having areas in which they needed to improve, the speeches did not have a strong foothold, but it could not be denied that their progress in the post-test was apparent. Following was an example of a student's academic progress illustrated in Figure 6.

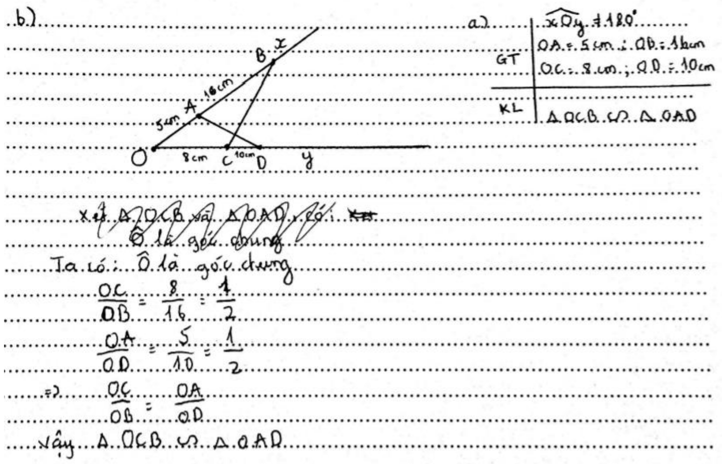

Figure 6. Student05's worksheet

(Translate in Figure 6: a) Summary: Given/Prove. b) We have *O* as the common angle. $\frac{OC}{OB}=\frac{8}{16}=\frac{1}{2}$; $\frac{OA}{OD}=\frac{5}{10}=\frac{1}{2}$, we infer: $\frac{OC}{OB}=\frac{OA}{OD}$, so $\Delta OCB \sim \Delta OAD$).

Discussion

Previously, students' mathematical communication skills were surveyed by tests to evaluate this capacity level of students in the experimental class to ensure objectivity and science in research, with initial comments on the level of student expression on mathematical communication skills. The four-step learning model introduced in the study promoted the improvement of students' mathematical abilities, most notably in mathematical communication skills; a similar learning model of discovery teaching was also implemented in the research of Juliarta and Landong (2020). Once the students had gone through the classroom activities, they were given an additional practice test to prepare them for the test they would take later. Several students' lesson analyses were utilized to summarize the experiment results, and the study of those results generated ideas and debates about the results. After conducting the study, it was found that the group agreed on the proposed teaching method, and it could be considered to be a legitimate teaching approach in the group. It was also found that the inquiry teaching methodology proved to be effective in helping students improve their ability to communicate mathematically. (Supiyanto et al., 2020).

In respect of knowledge, the students achieved the lesson's goal; they understood the concept of two congruent triangles and the congruent cases of two triangles (Alexander, 2018). In terms of skills, students applied the congruent triangle theorems to solve mathematical problems such as proving two congruent triangles; find the length of the line; calculate uniformity ratio, the ratio of the perimeter, ratio of the area; calculate the perimeter, area of the triangle; prove an identity. Additionally, the learners were able to apply real-world problem solving with congruent triangles to expand their understanding. During this time, they demonstrated a higher-than-average aptitude for learning and were too eager and excited when discovering the solution. While simultaneously having confidence in their determination to solve the exercises correctly, they also had high degrees of confidence in solving more advanced manipulation activities.

On the other hand, students demonstrated a too high level of mathematical communication system classifications, including understanding questions and requests, understanding the system of questions, and applying them to produce answers. Besides, the student developed various other capabilities included public speaking skills and a breadth of math topics and concepts that they explored through group interactions and communicating to their peers and class representatives. Another result that should also be taken into account is that visual and symbolic representations also contribute to a student's successful problem solving, where the symbolic representation participates in the problem-solving stages such as writing topics and writing conclusions (Utomo & Syarifah, 2021). Regarding the efficacy of their performance (Rahmi et al., 2017), there was a notable improvement in their work, as well, because they employed natural language mixed with mathematical jargon to apply their findings to practical use problems. Furthermore, another remarkable finding was that students with better mathematical communication skills demonstrated higher levels of performance in the proof of a mathematical theorem, and this result was consistent with the findings of the study of Salsabila (2019). It is possible to ascertain that good communication skills enhance the ability of students to perform well in their use of mathematical argumentation (Zhou et al., 2021).

This type of experimentation has also drawn out some potentially significant implications. In addition to these things, the teacher also acquired skills, including classroom organization and management, and the design and implementation of the lesson plan to enhance students' communication skills in mathematics. They also examined learners' capacity to speak mathematically and designed a lesson plan to improve this capability. Additionally, the students practiced more specialized communication skills, such as solving complex problems, making comparisons, and logical explanations for mathematical concepts. To significantly expand on this, this type of instruction process aimed explicitly at developing mathematical communication skills for students in an experimental class on congruent triangles can also be used to teach other math topics. After expanding from there, the teacher can instruct students on how to compare the features of geometries (Leung et al., 2014).

Conclusion

Mathematics is important in high schools, and the language of mathematics education is prominent in them. Talking about mathematical characteristics, along with the abstraction of mathematical objects, proof and discovery methods, discoveries in mathematics, people pay special attention to the language of mathematics. Research has shown the importance of using mathematical language to mathematical growth in children's thinking. Today, sign language, formalized language, has become a feature of modern mathematical thinking. In every math lesson, students go through an expansion in which they must first express ideas in an expressive language and then have practical fluency. Initial studies show that students still have many difficulties participating in communication and presenting mathematical content by themselves. Students' ability to speak and write math is still limited. Students are confused when presented with visual representation problems because they use only arithmetic representations during mathematics training, so they have trouble finding mathematical solutions in practice. By implication, it can be inferred that the ability to express mathematical ideas effectively in writing has a remarkable impact on the students' ability to construct valid proofs; and perhaps may cause difficulties (Salsabila, 2019).

Suppose teachers do not have effective measures to organize students to participate in learning activities in general, math communication activities. In that case, it is reasonable to infer that students lack initiative rather than confidence, lack of confidence, an uncomfortable learning environment, and motivation to engage in learning activities. Students lack flexibility in applying mathematics to solve real-life problems posed. The construction and organization of learning situations for students to engage in mathematical communication is not only a condition to stimulate the above activities but also contributes to clarifying the innovative orientation of teaching according to the development of mathematical competencies for learners, improving students' accountability and activeness, proactiveness in building mathematical understanding, building up their solid knowledge, forming and developing ability to connect math study with practice. In the context of renovating high school mathematics education, building measures to foster mathematical communication competencies for students in math teaching becomes more and more necessary towards the formation and development of competencies and quality for learners.

Recommendations

Many good recommendations are given for teachers who need useful materials to teach students about math communication skills. Firstly, the teachers need to strengthen listening comprehension, reading activities (texts, models, diagrams, drawings), and take notes (listening comprehension content, reading comprehension) in the mathematical language. Secondly, teachers instruct students to create speaking or writing mathematical tasks in teaching mathematical concepts, theorems, rules, and methods. Thirdly, the teachers help students construct, select, and assemble various mathematical communication activities to solve mathematical situations. Next, some interactive learning activities (in a group, in pairs, or in-class discussion) (Paruntu et al., 2018) need to perform diverse learning tasks in terms of solutions, with practical elements, with many suitable representations with students in awareness, practice, memorization, and mathematical communication. Furthermore, mathematics teachers should use contextual learning as an additional mathematics learning method to significantly improve their mathematics communication skills (Hutapea et al., 2019; Indah Nartani et al., 2015).

Furthermore, visual representations help students express mathematical ideas effectively (Yenny, 2016). Providing an ideal platform for people to display their abilities, an event with a strong emphasis on performance will allow them to build new mathematical knowledge. When students can see the visual representations, they can learn math better, thanks to the enhanced learning environment. There are well-documented claims that using alternative representations enables students to recognize and approach the nature of the problem, leading to their development of a solution to the problem. The classroom's overall arrangement can help encourage mathematical communication if cooperation is brought to the forefront in lessons with conflicting knowledge, which means one of two things:

teamwork in class or the knowledge presented was out of date. When working in groups, they communicate their ideas and express them by writing on paper, verbally. When they express that idea, they will use their symbols such as diagrams, drawings, letters, symbols they use mathematical representations.

Additionally, students must be aware of the importance of good mathematical communication skills for learning mathematics because the capacity of good mathematical communication encourages positive learning outcomes in mathematics, boosts logical thinking abilities, and establishes correct communication methods. This approach develops and reinforces the capacity mentioned above, which is focused on the learners and results in positivity, initiative, self-esteem, and a high degree of confidence in their capacity to learn new knowledge for the rest of their lives. The most significant use of students' math learning is gained when they are allowed to identify relevant solutions to problems and the chance to use their teachers' assistance. As a specific instance, when students solve problems, they frequently brainstorm several approaches to arrive at a solution. When students are uncertain about mathematics, they look for more information to help them better understand the problem; simultaneously, they try to apply new mathematics concepts by relating them to their prior knowledge. This result shows mathematical thinking in dealing with mathematical problems mentioned in the research of Utomo and Syarifah (2021). Hence, learning how to solve problems on their own is accomplished by engaging in independent work. For other researchers, we encourage the development of learning kits that can assist in fostering students' understanding of how to communicate mathematically. Furthermore, it is also recommended that new studies be conducted to determine whether or not it is related to students' mathematical capacities for communication and deduce mathematical proofs.

Limitations

Because this was a case study, only 40 students in grade 8 were observed. The ability and willingness of subjects to learn were drastically different between genders, and they could be split up along their learning style. Furthermore, research only focused on exploiting and using the language of mathematics, including mathematical symbols, terms, to foster students' mathematical communication competencies in teaching mathematics in class 8.

Authorship Contribution Statement

Bui Phuong Uyen: Drafting manuscript, critical revision of manuscript, supervision, final approval. Duong Huu Tong: Conceptualization, design, writing, editing/review. Nguyen Thi Bich Tram: Data acquisition, data analysis, statistical analysis.

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